On Sellers' Collusion in E-Commerce Marketplaces *

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Abstract

Motivated by a recent competition policy debate on retailers' collusion in online marketplaces, this paper studies a simple model to shed light on the competitive and welfare effects of this conduct. I find that, when retailers sell their products through a monopolistic e-commerce platform, consumers are not necessarily harmed by their collusive behaviour. Specifically, if the platform adopts the agency model and is vertically integrated (i.e., sells a private label in competition with third-party sellers), a cartel between third-party sellers induces it to charge them lower fees and to set a lower price for its private label. As a consequence, when products are sufficiently homogeneous, also the cartel members charge lower prices compared to the non-cooperative equilibrium, and collusion benefits consumers and increases total welfare. Notably, these results hold even though the platform has all the bargaining power *vis-à-vis* (competing or colluding) third-party sellers, and they collude explicitly.

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1 Introduction

It is widely recognized that the recent growth of e-commerce[1] has the potential to increase competition within retail markets, to enhance consumer choice, and to foster and facilitate innovation in product distribution (e.g., OECD, 2019). However, competition authorities on both sides of the Atlantic, as well as legal and economics scholars, have recently expressed concerns that competition in online marketplaces is likely to enlarge the scope for anti-competitive practices by sellers. In fact, the greater price transparency which characterizes online marketplaces allows retailers to track more effectively the prices charged by their rivals, which may facilitate collusion, whether explicit or tacit (OECD, 2019).

In particular, whether and to what extent the diffusion of algorithmic pricing among e-retailers[2] fosters collusion is widely debated (see, e.g., Harrington, 2018; Calvano et al., 2019, and references therein). In a recent antitrust case,[3] the CMA successfully challenged a pricing software that was allegedly designed to coordinate the price of posters by multiple online sellers. In brief, Trod Limited has admitted agreeing with one of its competing online sellers (GB eye Limited) that they would not undercut each other's prices for posters and frames sold on Amazon's UK website. Such agreement was implemented by using algorithmic pricing software which each of the parties configured to give effect to the illegal cartel.[4] In this case, as evidence of price-fixing existed, such collusion fell within the *per se* prohibition of cartels. However, the use of pricing algorithms may also facilitate tacit collusion: the experimental evidence provided by Klein (2019) and Calvano et al. (2020,2021) shows that, indeed, relatively simple pricing algorithms systematically *learn to play* collusive strategies,[5] which is confirmed by the empirical evidence in Assad et al. (2020). In this case, as argued by Harrington (2018), this form of collusion is not a violation of US or E.U. antitrust laws. On this ground, he advocates a *per se* prohibition on certain pricing algorithms that support supra-competitive prices.

However, Calvano et al. (2019) argue that an outright prohibition of algorithmic pricing is unlikely to be optimal, as there is a wide consensus that pricing algorithms may deliver significant efficiency gains by allowing more efficient pricing. Indeed, game theoretic models by Miklós-Thal and Tucker (2019) and O'Connor and Wilson (2020) show that the employ of algorithmic pricing tools which allow a better demand forecasting has ambiguous effects on the sustainability of collusion, and may also increase consumer surplus.[6]

[1] In 2019, retail e-commerce sales amounted to 3.53 trillion US dollars worldwide and e-retail revenues are projected to grow to 6.54 trillion US dollars in 2022. Online shopping is one of the most popular online activities worldwide. See, e.g., https://www.statista.com/statistics/379046/worldwide-retail-e-commerce-sales/.

[2] For evidence that a significant fraction of sellers in a large online marketplace (namely, Amazon US) adopted algorithmic pricing, see, e.g., Chen et al. (2016).

[3] CMA case 2015 n. 50223. The same conduct was successfully challenged also in the US. See: https://www.justice.gov/atr/case-document/file/628891/download.

[4] Notably, Amazon itself was not involved in the cartel and has not been investigated by the CMA. For further details, see: https://www.gov.uk/government/news/online-seller-admits-breaking-competition-law. After that case, the CMA has warned online traders against price fixing. See: https://www.theguardian.com/business/2016/nov/07/online-sellers-price-fixing-competition-and-markets-authority-amazon.

[5] See also Johnson et al. (2020), who however identify platform-design decisions who can benefit consumers even when algorithmic collusion might otherwise emerge, and also raise the platform's profits under some circumstances.

[6] The main insight is that better demand forecasting increases both the payoff from colluding and from deviating. On top of this, consumers also benefit from algorithmic pricing as prices are consistent with demand conditions at any

Moreover, algorithmic pricing is often employed by e-retailers selling on large online marketplaces owned by platforms such as Amazon or Alibaba (see, e.g., Chen et al., 2016). As these platforms are *digital gatekeepers*[7] — i.e., control access by third-party sellers to their users (Alexiadis and de Streel, 2020) — they can charge relatively large fees to e-retailers, which ultimately are passed on to consumers. Hence, it can be argued that allowing sellers to collude could increase their bargaining power *vis-à-vis* powerful e-commerce platforms, thereby enabling them to offset platforms' ability to extract a large share of their profits through fees, which may benefit final consumers as well (see, e.g., Kirkwood, 2014, and references therein). This argument closely resembles the *countervailing buyer power hypothesis* as an efficiency defence for consolidation in the retailing sector (Galbraith, 1952), which however was recognized by the competent authority in only one collusion case so far.[8]

Motivated by the outlined competition policy debate, this short paper proposes a first formal model to shed light on the competitive and welfare effects of sellers' collusion in e-commerce marketplaces owned by a gatekeeper platform.

Specifically, I consider two competing retailers selling through an e-commerce platform. The platform adopts the agency business model (Johnson, 2017) — i.e., it charges sellers per-unit or ad-valorem fees. In addition, and crucially, the platform is vertically integrated — i.e., commercializes a private label in competition with third-party sellers. Both assumptions are consistent with the real-world practices of large e-commerce platforms such as Amazon, Walmart, Apple's Appstore and Google's Playstore: see Etro (2020), Hagiu et al. (2020), and Padilla et al. (2020a,b).[9] Moreover, to identify the welfare effects of collusion in the starkest possible way, I assume right away that colluding retailers charge the prices which maximize their joint profit (explicit collusion).

In this environment, I show that allowing third-party sellers to collude may increase consumer surplus and total welfare. These results are driven by the incentives of the platform to cut its fees, as well as the price of its private label, in response to the third-party sellers' cartel. Importantly, rather than being driven by explicit countervailing buyer power forces, as P is assumed to have all the bargaining power *vis-à-vis* (competing or colluding) third-party sellers, these results emerge as the platform itself recognizes that the cartel, by softening product market competition, would lead to excessively high retail prices, which would inefficiently reduce sales from the platform's viewpoint. The platform is thus forced to lower its fees, so as to mitigate the double marginalization problem arising in the industry given the linear (or ad-valorem) structure of the fees.

Consequently, since distributing third-party sellers' products is less profitable when they collude,

period. However, Harrington (2020) finds that a pricing algorithm sold by a third-party is always designed so as to soften competition (i.e., it leads to higher prices), and this anticompetitive effect does not require multiple competitors to adopt it. Notably, all these papers take the marketplace environment as exogenous — i.e., unlike in this work, they abstract from considering a strategic e-commerce platform owning the marketplace in which sellers' competition takes place.

[7] Evidence that nearly half of US internet users start product searches on Amazon.com can be found at: https://www.emarketer.com/content/more-product-searches-start-on-amazon.

[8] Namely, Balmoral Cinema, Inc. v. Allied Artists Pictures Corp..
See: https://casetext.com/case/balmoral-cinema-v-allied-artists-pictures.

[9] Etro (2020) and Hagiu et al. (2020) focus on the welfare implications of allowing these e-commerce platforms to sell private labels on their marketplaces. Padilla et al. (2020a) examine the circumstances under which a platform can use self-preferencing to foreclose third-party sellers, whereas Padilla et al. (2020b) investigate the platforms' incentives to collect consumer data. Unlike the present paper, these articles focus on platforms' strategies and their welfare implications under the assumption of sellers' competition in the marketplace.

the platform optimally lowers also the price of its private label, so as to steer consumers towards its relatively more profitable business. Being confronted with lower fees and expecting a more aggressive competition from the platform in the product market, the cartel ends up setting lower retail prices than competing retailers when products are relatively homogeneous — i.e., when price competition between the cartel members and the vertically integrated platform is fierce. Whenever this is the case, consumers are unambiguously better off under collusion (as all products are available at a lower price), and allowing collusion is optimal also from a total welfare standpoint — i.e., consumers' and third-party sellers' benefits from collusion outweigh the platform's losses.

These results suggest that an antitrust policy based on a *per se* prohibition of all forms of explicit or tacit collusion among sellers in e-commerce marketplaces may harm consumers, even when the efficiencies driven by sellers' countervailing power *vis-à-vis* powerful gatekeeper platforms, or brought up by the algorithmic pricing tools they employ to collude, are not of first-order importance.

The remainder of the paper is organized as follows. The baseline model is set-up and analysed in Section 2 and 3, respectively. Section 4 discusses the robustness of the results with respect to the model assumptions. Section 5 concludes. Additional technical material is in the Appendix.

2 Model

Two retailers R_i, $i = A, B$, sell their products through an e-commerce platform P. In addition to hosting these third part-sellers, P also owns (and is vertically integrated with) a retail unit that distributes a private label in competition with them.[10] The platform adopts the so called *agency model* of business (e.g., Johnson, 2017). Specifically, it charges a fee f_i to R_i for each unit sold through its website. Following a recent literature (e.g., Etro, 2020; Padilla et al., 2020a,b), I assume throughout that consumers can purchase firms' products only through P's website — i.e., P is a *gatekeeper platform* (Alexiadis and de Streel, 2020) — or, more generally, the considered firms compete with each other only on the e-commerce marketplace owned by P.[11]

The products sold by the three firms are imperfect substitutes and (symmetrically) horizontally differentiated. Specifically, given retail prices p_A, p_B and p_P, consumers' demand for product $h = A, B, P$ is given by[12]

$$D^h\left(p_h, \sum_{k=A,B,P;\ k\neq h} p_k\right) \triangleq \frac{1}{(1-\gamma)(1+2\gamma)}\left(1-\gamma-(1+\gamma)p_h+\gamma \sum_{k=A,B,P;\ k\neq h} p_k\right), \tag{1}$$

where $\gamma \in (0,1)$ is an inverse measure of product differentiation. In what follows, let

$$D_1^h(\cdot) \triangleq \frac{\partial D^h(\cdot)}{\partial p_h} = -\frac{1+\gamma}{(1-\gamma)(1+2\gamma)} < 0, \quad D_2^h(\cdot) \triangleq \frac{\partial D^h(\cdot)}{\partial p_k} = \frac{\gamma}{(1-\gamma)(1+2\gamma)} \in (0, |D_1^h(\cdot)|).$$

[10] The analysis is unchanged if, rather than being vertically integrated, P purchases the products from a competitive fringe of suppliers — i.e., if it acts as a first-party retailer.

[11] If this is not the case, then multimarket contacts *à la* Bernheim and Whinston (1990) are more likely to make collusion (overall) anti-competitive.

[12] This demand system is derived from a linear-quadratic specification of a representative consumer's utility *à la* Singh and Vives (1984): see Appendix B.

All firms' production costs are linear and normalized to zero. Hence, for $i, j = A, B, j \neq i$, each R_i's profit is given by

$$\pi_{R_i} \triangleq (p_i - f_i) D^i\big(p_i, p_j + p_P\big),$$

whereas P's profit, normalizing to zero the cost of operating the marketplace, is

$$\pi_P \triangleq \sum_{i=A,B} f_i D^i\big(p_i, p_j + p_P\big) + p_P D^P\big(p_P, p_A + p_B\big).$$

The outside option value of each R_i, when it does not distribute its products through P's website, is also normalized to zero.

Third-party sellers may either act non-cooperatively (in which case, each R_i maximizes π_{R_i}) or, if collusion is allowed, they can form a cartel and set prices so as to maximize their joint profit. Thus, to analyse the competitive effects of collusion in the starkest way, I consider explicit collusion.[13]

Specifically, the timing of the game is as follows:

$t = 0$ Third-party sellers decide whether to form a cartel, if collusion is allowed;

$t = 1$ P bargains with competing third-party sellers or with the cartel over unit fees (more below);

$t = 2$ Firms active on the platform (i.e., third-party sellers who accepted P's offer and P itself) set (retail) prices.

Following the literature (e.g., Johansen and Vergé, 2017; Bisceglia et al., 2020), I assume that, absent collusion, P simultaneously and bilaterally bargains with each R_i over f_i, and the solution concept is Contract Equilibrium (Crémer and Riordan, 1987; Horn and Wolinsky, 1988) — i.e., when agreeing on f_i, both P and R_i believe that the equilibrium unit fee is agreed upon between P and the other third-party seller R_j.[14] Moreover, contracts are secret and interim unobservable — i.e., when choosing p_i, R_i still believes that the R_j is facing the equilibrium unit fee. When, instead, third-party sellers collude, the cartel members jointly bargain with P over unit fees. In this case, Contract Equilibria clearly coincide with Subgame Perfect Nash Equilibria (SPNE).

Notice that, even though, for the sake of exposition, I assume that P knows whether or not the cartel is in place, since third-party sellers always benefit from colluding, one can equivalently assume that P correctly forecasts that they form a cartel (or that they are able to collude through algorithmic pricing) whenever this is not forbidden, and then it separately bargains with them, exactly as it does absent collusion. Then, the colluding third-party sellers maximize their joint profit given the negotiated contracts.

Lastly, I assume that P makes take-it-or-leave-it (TIOLI) offers to (competing or colluding) third-party sellers. Notably, this assumption rules out that the pro-competitive effects of third-party sellers' collusion can be driven by an increase in their bargaining power *vis-à-vis* P.[15]

[13] Thus, unlike in Miklós-Thal and Tucker (2019) and O'Connor and Wilson (2020), sellers do not use algorithmic pricing tools, which also entail more efficient pricing decisions, to (tacitly) collude.

[14] As shown by Rey and Vergé (2020), any Contract Equilibrium is a Sequential Equilibrium of a game in which firms involved in multiple bargaining processes (in this model, P) delegate the negotiations to partner-specific agents.

[15] Indeed, as long as allocating more bargaining power to third-party sellers translates into lower unit fees, their collusion is more likely to improve consumer surplus by reducing the extent of double marginalization arising in the industry: a pro-competitive effect similar to countervailing buyer power (see, e.g., Iozzi and Valletti, 2014, and references therein).

3 Analysis

In this section, I derive the equilibrium fees and prices of the game in which third-party sellers act non-cooperatively (hereafter, *non-cooperative equilibrium*), denoted by (f^*, p_P^*, p^*), and in which they form a cartel (*collusive equilibrium*), denoted by (f^c, p_P^c, p^c). To simplify the exposition, I thus directly restrict attention to symmetric equilibria in which P offers the same fee to both third-party sellers, who then set the same retail price. However, it can be easily shown that the restriction to symmetric equilibria is without loss of generality under the demand system (1). Comparing the two equilibria then allows to establish the competitive and welfare effects of collusion.

3.1 Non-Cooperative Equilibrium

When third-party sellers compete with each other, at stage $t = 2$, upon observing the unit fee f_i, each R_i solves

$$\max_{p_i} \, (p_i - f_i) D^i\big(p_i,\ p_j^e + p_P^e\big),$$

where p_j^e and p_P^e are the prices that R_i expects to be set by R_j and P, respectively. As contracts are secret and interim unobservable, R_i believes that $f_j^e = f^*$, so that R_j is expected to set the equilibrium price (i.e., $p_j^e = p^*$), and P to optimally set p_P given the fees (f_i, f^*).

The standard first-order condition (hereafter, FOC) of this problem,[16]

$$D^i(\cdot) + (p_i - f_i) D_1^i(\cdot) = 0, \tag{2}$$

which trades off the gain on inframarginal consumers and the demand reduction following an increase in p_i, yields R_i's strategy, which is increasing in the unit distribution cost (i.e., f_i) and, by strategic complementarity among prices, in p_j^e and p_P^e as well.[17]

Simultaneously, P, who of course knows both f_A and f_B it offered to third-party sellers, chooses the price of its private label solving

$$\max_{p_P} \sum_{i=A,B} f_i D^i\big(p_i^e, p_j^e + p_P\big) + p_P D^P\big(p_P, p_A^e + p_B^e\big),$$

where p_i^e, $i = A, B$, is the price optimally set by R_i given the offer f_i made by P in the previous stage. The FOC of this problem can be written as follows:

$$\underbrace{\sum_{i=A,B} f_i D_2^i(\cdot)}_{\text{Third-Parties Externality } (+)} + \underbrace{D^P(\cdot) + p_P D_1^P(\cdot)}_{\text{Oligopoly Rule}} = 0. \tag{3}$$

In words, when setting p_P, besides taking into account the standard trade-off of a single-product

[16] It can be immediately checked that the second-order conditions of all problems considered in the analysis are satisfied for all $\gamma \in (0, 1)$, though they are omitted for brevity.

[17] To see this, notice that the left-hand side of (2) is increasing in f_i, p_j^e and p_P^e (as $D_1^i(\cdot) < 0 < D_2^i(\cdot)$), whereas it must be decreasing in p_i at the optimum (by the second-order condition). Using the implicit function theorem immediately yields these comparative statics results.

oligopolist, P also internalizes the positive effect that a larger price of its private label has on the demand of third-party sellers, hence on its revenues from their fees. As a consequence, the price of its private label is larger than the one it would set absent revenues from third-party sellers, and even more so when fees are larger, so that distributing third-party sellers' products is more profitable. In addition, larger fees translate into a larger p_P also because they imply larger (expected) retail prices by third-party sellers, and prices are strategic complements.[18]

Moving backward to stage $t = 1$, when negotiations over f_i take place, by the Contract Equilibrium approach, both P and R_i believe that $f_j^e = f^*$, whereby $p_j^e = p^*$. Specifically, P's problem *vis-à-vis* R_i can be written as follows:

$$\max_{f_i} f_i D^i\big(p_i^*(f_i, f^*), p^* + p_P^*(f_i, f^*)\big) + f^* D^j\big(p^*, p_i^*(f_i, f^*) + p_P^*(f_i, f^*)\big) + p_P^*(f_i, f^*) D^P\big(p_P^*(f_i, f^*), p_i^*(f_i, f^*) + p^*\big),$$

where $p_i^*(f_i, f^*)$ and $p_P^*(f_i, f^*)$ are the retail prices optimally charged by R_i and P when fees are set at f_i and $f_j = f^*$ — i.e., the solutions to the system of FOCs (2) and (3), under their common belief $f_j^e = f^*$ and $p_j^e = p^*$ (see Appendix A).

The FOC of this problem writes as

$$\underbrace{D^i(\cdot) + f_i\left(D_1^i(\cdot)\frac{\partial p_i^*(\cdot)}{\partial f_i} + D_2^i(\cdot)\frac{\partial p_P^*(\cdot)}{\partial f_i}\right)}_{\text{Revenues from } R_i} + \underbrace{f^* D_2^j(\cdot)\left(\frac{\partial p_i^*(\cdot)}{\partial f_i} + \frac{\partial p_P^*(\cdot)}{\partial f_i}\right)}_{\text{Revenues from } R_j \ (+)} +$$

$$+ \underbrace{\frac{\partial p_P^*(\cdot)}{\partial f_i} D^P(\cdot) + p_P^*(\cdot)\left(D_1^P(\cdot)\frac{\partial p_P^*(\cdot)}{\partial f_i} + D_2^P(\cdot)\frac{\partial p_i^*(\cdot)}{\partial f_i}\right)}_{\text{Revenues from Private Label}} = 0. \qquad (4)$$

In words, P takes into account that, by charging a larger f_i, R_i is induced to set a larger price p_i, and, by the mechanisms detailed above, P itself will optimally increase its price p_P (even though to a lesser extent than R_i, in order not to lose too many market shares at the benefit of R_j, who remains under the equilibrium contract).[19] As a result, an increase in f_i induces a drop in R_i's and P's demand, and an increase in R_j's sales (see Appendix A). Hence, by offering a larger f_i: *(i)* P collects higher unit fees from R_i, but on a lower amount of sales; *(ii)* it obtains larger revenues from R_j, as the latter's contract and behaviour is fixed, thereby its sales increase; and *(iii)* it sells lower quantities of its private label at a larger price.

Solving the FOC (4) and imposing symmetry — i.e., $f_i = f^*$ and $p^* = p_i(f^*, f^*)$ — yields the equilibrium fee and price set by third-party sellers, from which also $p_P^* = p_P(f^*, f^*)$ can be easily computed.[20]

[18] To see that p_P is increasing in the fees and third-party sellers' expected prices notice that the left-hand side of (3) is increasing in $f_i + f_j$ and $p_A^e + p_B^e$, and decreasing in p_P (by the second-order condition) at the optimum. The results then immediately follow from the implicit function theorem.

[19] That is, $\frac{\partial p_i^*(\cdot)}{\partial f_i} > \frac{\partial p_P^*(\cdot)}{\partial f_i} > 0$ (see Appendix A).

[20] Notably, this Contract Equilibrium coincides with the Perfect Bayesian Equilibrium (when it exists) of the game in which P simultaneously sets f_A and f_B, and each R_i holds *passive beliefs* about the contract f_j offered to R_j (see Rey and Vergé, 2004).

3.2 Collusive Equilibrium

When third-party sellers form a cartel (i.e., choose prices so as to maximize their joint profit), they are assumed to jointly bargain with P. To ease exposition and without loss of generality, suppose that P offers a fee f which applies to both third-party sellers.[21]

Thus, at stage $t = 2$, the cartel solves

$$\max_{p_A, p_B} \sum_{i=A,B} (p_i - f) D^i\big(p_i, p_j + p_P^e\big),$$

where p_P^e is the optimal price of P's private label (that the cartel expects to be set by P) given the observed f. The FOC with respect to p_i, $i = A, B$, writes as

$$\underbrace{D^i(\cdot) + (p_i - f) D_1^i(\cdot)}_{\text{(Non-Cooperative) Oligopoly Rule}} + \underbrace{(p_j - f) D_2^j(\cdot)}_{\text{Externality on } R_j\text{'s profit } (+)} = 0. \tag{5}$$

In words, when setting p_i, the cartel takes into account not only the standard trade-off between an increase in margins and a drop in R_i's demand, but also internalizes the positive effect of a larger p_i on the demand (hence, profit) of the other cartel member R_j. Of course, other things being equal (in particular, if $f = f^*$ and $p_P^e = p_P^*$) this implies that cartel members have stronger incentives to raise prices compared to competing retailers: the standard anti-competitive effect of collusion.[22]

Moreover, this anti-competitive force is magnified by P's behaviour at the retail pricing stage. To see this, notice that P's problem in $t = 2$ is as in Section 3.1. Hence, if $f_A = f_B = f^*$, since prices are strategic complements, anticipating a larger retail price by third-party sellers, P would optimally increase its price compared to the non-cooperative equilibrium so as to take profits from the softer product market competition.[23] In turn, again by strategic complementarity between retail prices, a larger expected price set by P would give the cartel members further incentives to raise their prices,[24] and so on. Therefore, if fees were set at the same level, regardless of collusion being in place or not, retail prices charged by all firms would be larger under collusion than in the non-cooperative

[21] Indeed, as the cartel members are fully symmetric, the unique Contract Equilibrium under collusion coincides with the unique SPNE of the game in which first P makes offers (f_A, f_B) to third-party sellers, who then jointly choose retail prices (simultaneously with P itself) to maximize their joint profit. Directly imposing that P is constrained to offer $f_A = f_B = f$ is just meant to make its problem in $t = 1$ formally analogous to the one it faces absent collusion (as there it optimizes with respect to one variable only).

[22] Formally,

$$\underbrace{D^i\big(p^*, p^* + p_P^*\big) + (p^* - f^*) D_1^i(\cdot)}_{=0 \text{ from } (2)} + \underbrace{(p^* - f^*) D_2^j(\cdot)}_{>0} > 0,$$

which, by the second-order conditions, immediately implies that the solutions to the FOCs (5) for $i = A, B$ must be larger than p^*.

[23] Formally, for $p_i^e > p^*$ (for some $i = A, B$),

$$\sum_{i=A,B} f^* D_2^i(\cdot) + D^P(p_P^*, p_A^e + p_B^e) + p_P^* D_1^P(\cdot) > 0,$$

as the left-hand side is equal to zero at the equilibrium without collusion, which immediately implies that the P's optimal price, which solves the FOC (3), must be larger than p_P^* whenever $p_i^e > p^*$ (for some $i = A, B$).

[24] This result immediately follows from the implicit function theorem, as the left-hand side of the FOC (5) is increasing in p_P^e.

equilibrium, thereby a cartel between third-party sellers would unambiguously harm consumers.

Moving backward to stage $t = 1$, when negotiating with the cartel a unit fee f, P solves:

$$\max_{f} \sum_{i=A,B} f D^i\big(p^c(f), p^c(f) + p^c_P(f)\big) + p^c_P(f) D^P\big(p^c_P(f), 2p^c(f)\big),$$

where $p^c(f)$ and $p^c_P(f)$ are the SPNE prices of third-party sellers and P, respectively, given the fees offered in $t = 1$ (see Appendix A). The FOC of this problem writes as

$$\underbrace{\sum_{i=A,B} D^i(\cdot) + f \sum_{i=A,B} \left[D^i_1(\cdot)\frac{\partial p^c(\cdot)}{\partial f} + D^i_2(\cdot)\left(\frac{\partial p^c(\cdot)}{\partial f} + \frac{\partial p^c_P(\cdot)}{\partial f}\right)\right] +}_{\text{Revenues from Third-Party Sellers}}$$

$$\underbrace{+ \frac{\partial p^c_P(\cdot)}{\partial f} D^P(\cdot) + p^c_P(\cdot)\left(D^P_1(\cdot)\frac{\partial p^c_P(\cdot)}{\partial f} + 2D^P_2(\cdot)\frac{\partial p^c(\cdot)}{\partial f}\right)}_{\text{Revenues from Private Label}} = 0. \qquad (6)$$

In words, when offering a fee f to the cartel, P takes into account that a larger value of f translates into larger retail prices of all products: P will optimally increase the price of its private label as distributing third-party sellers' products becomes more profitable, and third-party sellers will do the same, and to a larger extent, because they face larger distribution costs.[25] As a consequence, unlike in the non-cooperative game, increasing f always causes a reduction of all firms' sales (see Appendix A).

3.3 Competitive and Welfare Analysis

Comparing the equilibrium prices in the two games yields the following results.

Proposition 1. *When third-party sellers collude, P optimally lowers both the unit fees ($f^c < f^*$) and the price of its private label ($p^c_P < p^*_P$). Third-party sellers charge a lower price in the collusive equilibrium if and only if products are sufficiently homogeneous — i.e., $p^c < p^*$ for $\gamma > \hat{\gamma}$, with $\hat{\gamma} \in (0, 1)$.*

The intuition behind the comparison of platform fees is as follows. Under sellers' competition, an increase in f_i, leading R_i to increase its retail price, boosts the demand of the other third-party seller, hence P's revenues from distributing R_j's products (as, by the Contract Equilibrium logic, R_j's fee and price are taken as given when negotiations over f_i take place). By contrast, when offering a contract to the cartel, P anticipates that an increase in f will cause a drop in both third-party sellers' sales. Moreover, the double marginalization problem, arising in the industry under unit platform fees, bites more when third-party sellers collude. This is because, under collusion, each R_i is not afraid of losing market shares at the benefit of R_j, which gives it stronger incentives to charge a larger mark-up or, equivalently, to pass on fees to consumers.[26] Taken together, these two observations imply that, under

[25] That is, $\frac{\partial p^c(\cdot)}{\partial f} > \frac{\partial p^c_P(\cdot)}{\partial f} > 0$ (see Appendix A).

[26] Indeed, third-party sellers' pass-through rate is always larger under collusion. Formally, $\frac{\partial p^c(\cdot)}{\partial f} > \frac{\partial p^*_i(\cdot)}{\partial f_i}$ (see Appendix A).

collusion, P anticipates that, by charging the unit fee f^* it sets in the non-cooperative equilibrium, third-party sellers would set excessively large prices, which would inefficiently reduce their sales, and accordingly result in lower revenues for P as a marketplace owner. As products are differentiated (i.e., consumers exhibit preferences for variety), these lower revenues cannot be fully compensated through the increase in the demand for P's private label. Hence, its optimal strategy consists in lowering unit fees in response to the third-party sellers' collusive behaviour — i.e., it optimally sets $f^c < f^*$.

Moreover, since lower fees imply that the profitability of hosting third-party sellers drops, P has stronger incentives to compete aggressively in the marketplace when third-party sellers collude, so to divert market shares towards the relatively more profitable business. As a result, $p_P^c < p_P^*$.

The comparison between the prices set by third-party sellers in the two equilibria is in general ambiguous. This is because, when they collude, on the one hand they can profitably increase their mark-ups (which, other things being equal, would imply $p^c > p^*$), but on the other hand they face lower distribution costs (which, other things being equal, would imply $p^c < p^*$). Importantly, their choices are also driven by the price of P's private label that they expect to be set in equilibrium: anticipating P's incentives to cut p_P under collusion (which follows from $f^c < f^*$), by the strategic complementarity between retail prices, colluding sellers have a further reason to charge a lower price. Indeed, this effect is more relevant when strategic complementarity forces are strong enough — i.e., competition is fierce (γ large). To put it differently, when products are relatively homogeneous, the presence of P's private label constitutes a strong competitive constraint on the cartel's choices. This implies that third-party sellers cannot increase too much their mark-ups under collusion, as otherwise they would lose too many market shares at the benefit of P's private label. As a result, since they face lower distribution costs, colluding sellers optimally reduce their prices compared to the non-cooperative equilibrium.

Yet, the elimination of competition among themselves and the lower fees offered them by P imply that third-party sellers charge higher mark-ups and are unambiguously better off under collusion than when they compete with each other (see Appendix A). As a consequence, when third-party sellers' explicit collusion is allowed by competition authorities, or they find other legal ways to implicitly collude (e.g., through the adoption of algorithmic pricing tools), they always find it optimal to do so. Then, a necessary condition in order for collusion to be pro-competitive is that P is able to observe (or correctly anticipate) their collusive behaviour (even if it does not take part to the cartel), and to adjust its fees and price accordingly: if this is not the case, then, as shown in Section 3.2, collusion always results in larger third-party sellers' mark-ups, hence prices.

Finally, from the price-effects of collusion detailed above, the following welfare results immediately follow.

Proposition 2. *There are thresholds $0 < \gamma^{CS} < \gamma^{TW} < \hat{\gamma}$ such that third-party sellers' collusion benefits consumers if and only if $\gamma > \gamma^{CS}$ and increases total welfare if and only if $\gamma > \gamma^{TW}$.*

Thus, allowing sellers to collude in e-commerce marketplaces in which the platform itself sells private labels may be desirable from a consumer surplus viewpoint. Notably, these results are obtained abstracting from the consideration that a sellers' cartel may have more bargaining power *vis-à-vis* powerful gatekeeper platforms. Rather than reflecting a different allocation of bargaining power, the lower unit fees charged by the platform are a by-product of its concerns about the excessive double marginalization problem induced by third-party sellers' collusion.[27]

However, in order for collusion to be pro-competitive, competition in the marketplace must be sufficiently intense. In particular, P's private label must be a relatively close substitute of third-party sellers' products. This is because, when cross-price elasticities are small, P has weaker incentives to reduce the price of its private label, and also third-party sellers are less concerned about P's pricing behaviour and thus less afraid of setting large prices. In these cases, the standard anti-competitive forces of collusion are bound to prevail.

In particular, for $\gamma < \gamma^{CS}$ the harm caused to consumers by the larger price of third-party sellers' products is not compensated by the availability of P's private label at a lower price, as products are rather differentiated, and two out of the three available varieties are sold at a higher price. By contrast, for $\gamma \in (\gamma^{CS}, \gamma^{TW})$ the cartel is beneficial to consumers, but it still decreases total welfare. This is due to the fact that the lower margins earned by P both as a marketplace and as a seller are not outweighed by the beneficial effect of the cartel on third-party sellers' profits and consumer surplus. However, since $\gamma^{TW} < \hat{\gamma}$, third-party sellers' collusion always increases total welfare when it lowers prices of all products in the marketplace.

4 Robustness and Discussion

4.1 Ad-valorem Fees

In the baseline model, following the literature (e.g., Boik and Corts, 2016; Johansen and Vergé, 2017; Mariotto and Verdier, 2020), I considered per-unit platform fees, which allowed to derive simple closed-form solutions for the equilibrium values. However, in the reality, most e-commerce platforms charge *ad-valorem fees*[28] (e.g., Johnson, 2017; Wang and Wright, 2017) — i.e., P collects $\phi_i p_i$, with $\phi_i \in [0, 1]$, for each unit sold by R_i (at price p_i).[29]

Indeed, considering ad-valorem fees magnifies the pro-competitive effects of third-party sellers' collusion (see Appendix C for the details). In particular, as in the baseline model, and for the same reasons, the presence of a cartel induces P to set lower fees and to reduce the price of its private label. However, under ad-valorem fees also third-party sellers always end up setting lower prices when they collude. The reason is that the anti-competitive effect of collusion is weaker when third-party sellers pay ad-valorem fees. This is because, unlike under unit fees, they do not fully appropriate any increase in the price they charge final consumers. As a consequence, the cartel has weaker incentives to increase prices under ad-valorem fees than under unit fees.

[27] In this respect, these results (and the intuition behind them) are close to those of recent papers showing how the welfare effect of information sharing among retailers (Gaudin, 2019), wholesale price-parity clauses (Bisceglia et al., 2020) and vertical contract disclosure (Bisceglia, 2020) depend on the extent of the double marginalization problem arising in vertical industries under linear contracts.

[28] See, e.g.: https://www.theverge.com/21445923/platform-fees-apps-games-business-marketplace-apple-google.

[29] Of course, to make things interesting in this setting, firms' (linear) production costs cannot be normalized to zero, as otherwise P would always optimally set $\phi_i = 1$ (for $i = A, B$) so as to extract all third-party sellers' profits.

4.2 Alternative Demand and Cost Functions

The foregoing analysis was carried out under a linear demand system *à la* Singh and Vives (1984). This linear specification of the model has the advantage of simple closed-form solutions, which cannot be obtained under more general demand systems, as P and colluding retailers are *de facto* multi-product firms, which greatly complicates the analysis. Moreover, it can be argued that the linear specification does not limit the generality of the results, as this paper aimed to derive possibility results, showing that the welfare effects of collusion may be positive or negative depending on a demand-specific parameter.[30]

Yet, the results of Section 3.3 seem qualitatively robust with respect to the demand specification. First, the qualitative results of Propositions 1 and 2 also hold under a linear demand specification *à la* Shubik and Levitan (1980): see Appendix G for a formal analysis. Second, the symmetric horizontal differentiation assumption is not crucial to the results: in Appendix D it is shown that, if the product sold by P is asymmetrically differentiated compared to the products sold by third-party sellers, then P still always lowers both fees and p_P in the collusive equilibrium, and third-party sellers' prices are lower under collusion if P's product is a relatively close substitute of both their products (so that the cartel fears competition from P's private label), even if third-party sellers' products are rather homogeneous (whereby collusion eliminates a strong competitive constraint).

Similarly, as far as the cost-symmetry assumption is concerned, in Appendix E it is shown that, if P suffers from a cost disadvantage *vis-à-vis* third-party sellers — i.e., it incurs a unit production cost $c_P > 0$ — then the exact same results of Proposition 1 hold. However, the threshold γ^{CS} above which collusion is pro-competitive is an increasing function of c_P. The reason is that, as c_P grows larger, other things being equal, the market share of P's private label drops (as P always sets a larger price compared to the baseline model in which $c_P = 0$). This implies that relatively more consumers purchase third-party sellers' products, which are more expensive in the collusive equilibrium for all $\gamma < \hat{\gamma}$. Nonetheless, $\gamma > \hat{\gamma}$ remains a sufficient condition in order for collusion to benefit consumers, as under this condition all products are available at a lower price in the collusive equilibrium.

4.3 Increased Competition in the Marketplace

One may wonder how the welfare results of Section 3.3 change when considering a larger number of third-party sellers in the marketplace. To make a case against collusion, I assume that the explicit collusive agreement always extends to all third-party sellers.

Appendix F shows that, extending to $N \geq 2$ symmetric third-party sellers the demand system *à la* Singh and Vives (1984) considered in the baseline model, the exact same results of Proposition 1 hold true for every N. However, a larger degree of product homogeneity is needed in order for collusion to be pro-competitive in less concentrated markets. This is because, as N grows larger, the market share of P's private label drops, which implies that the share of consumers who are harmed by third-party sellers' collusion for all $\gamma < \hat{\gamma}$ is larger in less concentrated marketplaces.

Indeed, the negative effect of N on the competitive effects of collusion is magnified under a demand

[30] For similar reasons, linear demand specifications have been often employed in the related literature (e.g., Iozzi and Valletti, 2014; Johansen and Vergé, 2017; Mariotto and Verdier, 2020).

system *à la* Shubik and Levitan (1980). In that case, while fees and P's private label's price are still always lower in the collusive equilibrium for all N, a larger degree of product substitutability is needed in order for $p^c < p^*$, as N grows larger (see Appendix G). The reason is that, since (in a symmetric equilibrium) aggregate demand does not depend on the number of firms (e.g., Motta, 2004), competition in the marketplace becomes much fiercer as N grows larger, which magnifies the anti-competitive effect of collusion.

These results, together with those discussed in the previous section, show that, in order for third-party sellers' collusion to be pro-competitive, the cartel must face fierce competition from P's private label. To put it differently, if P suffers a large comparative disadvantage *vis-à-vis* third-party sellers or, for other reasons, has negligible market shares, then its more limited ability to divert business towards its private label is not effective in mitigating the excessive marginalization problem brought up by the third-party sellers' cartel, thereby making the standard anti-competitive effects of collusion more likely to prevail.

4.4 Alternative Timing and Solution Concept

In the baseline model, following the literature and to ease exposition, I solved for the Contract Equilibrium of a game in which P first makes offers to third-party sellers and then sets the price p_P in competition with the sellers themselves. However, the main results are qualitatively robust with respect to the timing and solution concept specification. Namely, consider the following alternative timing:[31]

$t = 1$ P simultaneously sets f_i, $i = A, B$, and p_P;

$t = 2$ Absent collusion, each R_i, $i = A, B$, chooses

$$p_i = \arg\max_{p_i} \, (p_i - f_i) D^i\big(p_i, p_j^e + p_P^e\big),$$

whereas the cartel sets

$$(p_A, p_B) = \arg\max_{p_A, p_B} \sum_{i=A,B} (p_i - f_i) D^i\big(p_i, p_j + p_P^e\big).$$

Solution concept is Perfect Bayesian Equilibrium (PBE), as now P simultaneously sets all prices. As for out-of-equilibrium beliefs, to make things more interesting,[32] I consider (affine) *wary beliefs* (see, e.g., Rey and Vergé, 2004; Gaudin, 2019; Bisceglia, 2020) — i.e., each R_i believes that P has optimally set the other prices (i.e., p_P, and also f_j absent collusion), given the observed offers (f_i, and also f_j under collusion).

This alternative specification of the model weakens the pro-competitive effect of third-party sellers' collusion. Specifically, collusion still lowers P's fees and price p_P, but it always leads to an increase

[31] Stage $t = 0$ is of course as in the baseline model.

[32] Indeed, under passive beliefs the results of Proposition 1 and 2 still hold, though the values of the thresholds slightly change compared to the baseline model with sequential P's choices. Details are omitted for brevity and available upon request.

in third-party sellers' prices. The intuition is as follows. Absent collusion, the double marginalization problem is more pronounced than in the baseline model. This is because, confronted with a larger fee f_i, unlike under passive beliefs (or Contract Equilibrium), R_i believes that also R_j faces a larger fee, hence is expected to set a larger p_j, which (by strategic complementarity) gives R_i more incentives to increase its price. Therefore, in this model collusion exacerbates double marginalization to a lesser extent compared to the model under passive beliefs, which in turn gives P weaker incentives to cut fees and compete more aggressively with colluding third-party sellers. As a consequence, $p^c > p^*$ for all γ. Yet, collusion is beneficial to consumers when products are sufficiently homogeneous, even though the threshold γ^{CS} above which collusion is pro-competitive is larger than in the base model. A detailed analysis is contained in Appendix H.

4.5 Platform's Business Model

The foregoing sections have shown the robustness of the main results of the paper with respect to several technical assumptions. Here instead I focus on the assumptions on P's business model which are crucial to the results.

First, P must adopt an agency business model. To see this, suppose that P is instead organized according to a wholesale (or resale) business model — i.e., it first purchases the products from third-party sellers (at wholesale prices set by the sellers themselves), and then sets the retail prices of all products. In this case, colluding third-party sellers always find it optimal to increase the wholesale prices charged to P, which P then passes on to consumers. Since (under linear demand) the price of P's private label is set at the same level regardless of third-party sellers' wholesale prices, it follows that their collusion is always anti-competitive under the wholesale business model.[33] A formal analysis is contained in Appendix I.

Second, P must be vertically integrated. Indeed, if only third-party sellers distribute their products in the online marketplace owned by P, then a cartel of all third-party sellers, or even of a subset of them, always damages consumers. This is because P has weaker incentives to lower its fees compared to the base model, as here lower fees cannot be used as a commitment device to set a lower price for the private label so to discipline colluding sellers. Then, since retail prices of all sellers, including those who do not take part to the cartel, are strategic complements, collusion unambiguously results in higher prices, absent the competitive constraint induced by P's private label: detailed proofs are in Appendix J. This finding is perfectly in line with the results summarized in Sections 4.2 and 4.3, which clearly pointed out that the pro-competitive effects of collusion crucially hinge on the assumption that the cartel faces tough competition from P's private label.

4.6 Model Applicability

The model analysed so far also applies to third-party sellers' concentration, rather than collusion, since the cartel behaves exactly as a multi-product merged entity. Hence, this paper suggests that horizontal mergers between third-party sellers (who optimally behave as multi-product entities following the

[33] Indeed, in this model the anti-competitive effect of collusion even strengthens when colluding sellers gain bargaining power *vis-à-vis* P, as *ceteris paribus* more powerful sellers are able to charge higher wholesale prices.

merger) may benefit consumers even absent efficiencies or an increase in the merged entity bargaining power *vis-à-vis* the vertically integrated (gatekeeper) platform.

Moreover, the same model can be applied also to industries organized with a wholesale business model, in which P is a monopolist manufacturer selling its product both through two competing, horizontally differentiated, retailers (supplied under linear contracts) and a direct sale channel.[34] Hence, this paper shows that retail mergers may be pro-competitive in the presence of supplier encroachment, even absent countervailing buyer power forces *à la* Galbraith (1952).[35]

5 Conclusion

Motivated by the growing concerns over sellers' collusion in online marketplaces (e.g., through the use of pricing algorithms), I have built a simple model to show that such a collusive behaviour is not bound to produce anti-competitive effects.

Specifically, when the gatekeeper platform owning the marketplace is vertically integrated — i.e., as it often happens in reality, sells a private label in competition with the colluding third-party sellers — collusion benefits consumers and increases total welfare when products are sufficiently homogeneous. This is because, since third-party sellers' collusion exacerbates the double marginalization problem arising under the unit (or ad-valorem) fees typically charged by e-commerce platforms, the platform itself finds it optimal to cut these fees so as to avoid an excessive drop in consumers' demand, and at the same time also reduces the price of its private label, so to steer consumers towards its relatively more profitable business. The lower fees, together with a more aggressive competition by the platform's private label, then lead colluding sellers to reduce their prices compared to the non-cooperative equilibrium, when product market competition is sufficiently intense.

As detailed above, these results hinge on a number of assumptions, which need not always be satisfied in reality. Among the crucial ones: *(i)* third-party sellers compete with each other only on the e-commerce platform's website; *(ii)* the platform adopts the agency business model, is vertically integrated, and, as a seller, is an effective competitor of the third-party sellers it hosts; *(iii)* the platform does not take part to the cartel, but observes (or correctly anticipates) collusion by third-party sellers, and adjusts its fees and price accordingly. Moreover, this static model (which takes the industry structure as given) neglects the effects of collusion on firms' dynamic incentives (e.g., to innovate, or to enter the marketplace itself), whose formal investigation is left for future research.

Yet, this paper suggests that *per se* illegality is unlikely to be the optimal competition policy standard for all tacit and explicit cases of collusion in e-commerce marketplaces, even when this collusive behaviour is ineffective in offsetting platforms' bargaining power and the algorithmic pricing tools which prompt collusion do not entail any consumer surplus enhancing efficiency in sellers' pricing.

[34]This *dual distribution* mode is rather common in many industries, and widely analysed in the literature (see Chiang et al., 2003, and Arya et al., 2007, among many others). For evidence of linear contracts being prevalent in several industries, see, e.g., Gaudin (2019).

[35]Notably, the welfare results of this model crucially hinge on the presence of a double marginalization problem in the industry, induced by linear contracts. While one might expect collusion to have more anti-competitive effects under more efficient non-linear contracts (e.g., two-part tariffs), a full-fledged analysis of the game under more complex contractual arrangements is outside the scope of this paper, as these contracts are rarely employed in the reality by e-commerce platforms organized according to an agency model.

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A Supplementary calculations

Non-cooperative Equilibrium. Solving the FOCs (2) and (3) yields, respectively:

$$p_i(f_i; p_j^e, p_P^e) = \frac{1 - \gamma + (1+\gamma)f_i + \gamma(p_j^e + p_P^e)}{2(1+\gamma)},$$

and

$$p_P(f_i, f_j; p_i^e, p_j^e) = \frac{1 - \gamma + \gamma(f_i + f_j + p_i^e + p_j^e)}{2(1+\gamma)}.$$

The prices $p_i(f_i, f^*)$ and $p_P(f_i, f^*)$ are obtained solving

$$\begin{cases} p_i = p_i(f_i; p^*, p^P) \\ p_P = p_P(f_i, f^*; p_i, p^*) \end{cases}$$

The solution is given by

$$p_i^*(f_i, f^*) = \frac{2 + \gamma - 3\gamma^2 + (2 + 4\gamma + 3\gamma^2)f_i + \gamma(2+3\gamma)p^* + \gamma^2(3p^* + f^*)}{(2+\gamma)(2+3\gamma)}$$

and

$$p_P^*(f_i, f^*) = \frac{3\gamma(1+\gamma)f_i + (2+3\gamma)(1 - \gamma(1-p^*)) + 2\gamma(1+\gamma)f^*}{(2+\gamma)(2+3\gamma)},$$

with[36]

$$\frac{\partial p_i^*(\cdot)}{\partial f_i} > \frac{\partial p_P^*(\cdot)}{\partial f_i} > 0,$$

for all $\gamma \in (0,1)$. Moreover,

$$\frac{\partial D^i(\cdot)}{\partial f_i} = D_1^i(\cdot)\frac{\partial p_i^*(\cdot)}{\partial f_i} + D_2^i(\cdot)\frac{\partial p_P^*(\cdot)}{\partial f_i} = -\frac{2(1+\gamma)(1+2\gamma)}{(1-\gamma)(1+2\gamma)(2+\gamma)(2+3\gamma)} < 0 \quad \forall\gamma \in (0,1),$$
$$\frac{\partial D^j(\cdot)}{\partial f_i} = D_2^j(\cdot)\left(\frac{\partial p_i^*(\cdot)}{\partial f_i} + \frac{\partial p_P^*(\cdot)}{\partial f_i}\right) = \frac{\gamma(2+7\gamma+6\gamma^2)}{(1-\gamma)(1+2\gamma)(2+\gamma)(2+3\gamma)} > 0 \quad \forall\gamma \in (0,1),$$

and

$$\frac{\partial D^P(\cdot)}{\partial f_i} = D_1^P(\cdot)\frac{\partial p_P^*(\cdot)}{\partial f_i} + D_2^P(\cdot)\frac{\partial p_i^*(\cdot)}{\partial f_i} = -\frac{\gamma(1+2\gamma)}{(1-\gamma)(1+2\gamma)(2+\gamma)(2+3\gamma)} < 0 \quad \forall\gamma \in (0,1).$$

Using these strategies, following the steps detailed in Section 3.1, one finds that, when third-party sellers act non-cooperatively, P charges them unit fees

$$f^* = \frac{(2+3\gamma)(4+6\gamma+3\gamma^2)}{2(2+\gamma)(4+9\gamma+6\gamma^2)},$$

[36] Indeed,

$$\frac{\partial p_i^*(\cdot)}{\partial f_i} - \frac{\partial p_P^*(\cdot)}{\partial f_i} = \frac{2+\gamma}{(2+\gamma)(2+3\gamma)} > 0 \quad \forall\gamma \in (0,1).$$

and sells its private label at price

$$p_P^* = \frac{1}{2} + \frac{2}{5(2+\gamma)} - \frac{2(2+\gamma)}{10(4+9\gamma+6\gamma^2)}.$$

Third-party sellers set price

$$p^* = \frac{1}{4} + \frac{1}{2+\gamma} - \frac{\gamma}{4(4+9\gamma+6\gamma^2)},$$

with $p^* > p_P^*$.

Collusive Equilibrium. Solving the system of FOCs (5) yields

$$p(f; p_P^e) = \frac{1}{2}(1-\gamma(1-p_P^e)+f).$$

Similar to the previous case, P's pricing strategy is as follows:

$$p_P(f; p^e) = \frac{1-\gamma+2\gamma(f+p^e)}{2(1+\gamma)}.$$

Using these two strategies it is easy to find the SPNE prices for any given fee f:

$$p^c(f) = \frac{2+\gamma-3\gamma^2+2(1+\gamma+\gamma^2)f}{4+2(2-\gamma)\gamma}, \quad p_P^c(f) = \frac{1-\gamma^2+3\gamma f}{2+(2-\gamma)\gamma},$$

with[37]

$$\frac{\partial p^c(\cdot)}{\partial f} > \frac{\partial p_P^c(\cdot)}{\partial f} > 0,$$

and[38]

$$\frac{\partial p^c(\cdot)}{\partial f} > \frac{\partial p_i^*(\cdot)}{\partial f_i} > 0,$$

for all $\gamma \in (0,1)$. Notice also that

$$\frac{\partial D^i(\cdot)}{\partial f} = D_1^i(\cdot)\frac{\partial p^c(\cdot)}{\partial f} + D_2^i(\cdot)\left(\frac{\partial p^c(\cdot)}{\partial f} + \frac{\partial p_P^c(\cdot)}{\partial f}\right) = \frac{1}{(1-\gamma)(1+2\gamma)}\left(\frac{3(1+\gamma)}{2+\gamma(2-\gamma)} - 2\right) < 0 \quad \forall \gamma \in (0,1),$$

and

$$\frac{\partial D^P(\cdot)}{\partial f} = D_1^P(\cdot)\frac{\partial p_P^c(\cdot)}{\partial f} + 2D_2^P(\cdot)\frac{\partial p^c(\cdot)}{\partial f} = -\frac{\gamma(1+\gamma-2\gamma^2)}{(1-\gamma)(1+2\gamma)(2+\gamma(2-\gamma))} < 0 \quad \forall \gamma \in (0,1).$$

[37]Indeed,

$$\frac{\partial p^c(\cdot)}{\partial f} - \frac{\partial p_P^c(\cdot)}{\partial f} = \frac{3}{2+\gamma(2-\gamma)} - 1 > 0 \quad \forall \gamma \in (0,1).$$

[38]Indeed,

$$\frac{\partial p^c(\cdot)}{\partial f} - \frac{\partial p_i^*(\cdot)}{\partial f_i} = \frac{3\gamma^2(1+\gamma)(1+2\gamma)}{(2+\gamma)(2+3\gamma)(2+(2-\gamma)\gamma)} > 0 \quad \forall \gamma \in (0,1).$$

Solving the FOC (6), using the SPNE strategies derived above, yields the equilibrium unit fee

$$f^c = \frac{4 + 4\gamma + \gamma^3}{2(2+\gamma)^2},$$

which, substituted into SPNE strategies, gives the equilibrium price of P's private label,

$$p_P^c = p_P^c(f^c) = \frac{1}{2} + \frac{\gamma(1-\gamma)}{4 + 4\gamma + \gamma^2},$$

and of third-party sellers products,

$$p^c = p^c(f^c) = \frac{3-\gamma}{4} + \frac{\gamma^2(1-\gamma)}{4(4 + 4\gamma + \gamma^2)},$$

with $p^c > p_P^c$.

B Proofs

Proof of Proposition 1. Comparing equilibrium fees and prices immediately yields

$$f^c - f^* = -\frac{\gamma(2 + 6\gamma + 7\gamma^2 - 3\gamma^4)}{(2+\gamma)^2(4 + 9\gamma + 6\gamma^2)} < 0 \quad \forall \gamma \in (0,1),$$

$$p_P^c - p_P^* = -\frac{\gamma^2(1 + 5\gamma + 6\gamma^2)}{(2+\gamma)^2(4 + 9\gamma + 6\gamma^2)} < 0 \quad \forall \gamma \in (0,1),$$

and[39]

$$p^c - p^* = -\frac{\gamma^2(1 + 5\gamma + 6\gamma^2)}{(2+\gamma)^2(4 + 9\gamma + 6\gamma^2)} < 0 \iff \gamma > \hat{\gamma},$$

with $\hat{\gamma} = \frac{10^{1/3}-1}{3} \simeq 0.38$. □

Proof of Proposition 2. Consumer surplus is given by the representative consumer's utility from which the demand system (1) is obtained:

$$U(\cdot) \triangleq \sum_{h=A,B,P} q_h - \frac{1}{2} \sum_{h=A,B,P} q_h^2 - \gamma \sum_{h,j=A,B,P;k\neq h} q_k q_h - \sum_{h=A,B,P} p_h q_h + M,$$

where $M > 0$ is the utility from income and q_h, $h = A, B, P$, the consumed quantities.

[39] Accordingly, third-party sellers' mark-ups are given by

$$m^* \triangleq p^* - f^* = \frac{(1-\gamma)(2 + 4\gamma + 3\gamma^2)}{(2+\gamma)(4 + 9\gamma + 6\gamma^2)}, \quad m^c \triangleq p^c - f^c = \frac{1-\gamma^3}{(2+\gamma)^2},$$

with

$$m^c - m^* = \frac{3\gamma(1-\gamma^2)(1 + 2\gamma(1+\gamma))}{(2+\gamma)^2(4 + 9\gamma + 6\gamma^2)} > 0,$$

for all $\gamma \in (0,1)$.

Substituting the equilibrium values yields

$$U^* = \frac{96 + 576\gamma + 1492\gamma^2 + 2156\gamma^3 + 1833\gamma^4 + 870\gamma^5 + 180\gamma^6}{8(2+\gamma)^2(1+2\gamma)(4+9\gamma+6\gamma^2)^2}$$

in the non-cooperative equilibrium, and

$$U^c = \frac{24 + 56\gamma + 68\gamma^2 + 5\gamma^3(12 + 5\gamma + 2\gamma^2)}{8(2+\gamma)^4(1+2\gamma)}$$

under collusion. Taking the difference gives

$$U^c - U^* = \frac{\gamma(-16 - 38\gamma + 126\gamma^2 + 739\gamma^3 + 1523\gamma^4 + 1710\gamma^5 + 1140\gamma^6 + 450\gamma^7 + 90\gamma^8)}{2(2+\gamma)^4(1+2\gamma)(4+9\gamma+6\gamma^2)^2} > 0 \iff \gamma > \gamma^{CS},$$

with $\gamma^{CS} \simeq 0.24 < \hat{\gamma}$.

Firms' profits in the non-cooperative equilibrium are given by

$$\pi_R^* = \frac{(1-\gamma^2)(2+\gamma(4+3\gamma))^2}{(2+\gamma)^2(1+2\gamma)(4+9\gamma+6\gamma^2)^2}, \quad \pi_P^* = \frac{(2+3\gamma)(64 + 304\gamma + 594\gamma^2 + 591\gamma^3 + 298\gamma^4 + 60\gamma^5)}{4(2+\gamma)^2(1+2\gamma)(4+9\gamma+6\gamma^2)^2},$$

whereas, when third-party sellers collude,[40]

$$\pi_R^c = \frac{(1-\gamma)(1+\gamma+\gamma^2)^2}{(2+\gamma)^4(1+2\gamma)}, \quad \pi_P^c = \frac{8+\gamma(12+5\gamma+2\gamma^2)}{4(2+\gamma)^2(1+2\gamma)}.$$

Total welfare in the two equilibria, defined as the unweighed sum of consumer surplus and industry profits ($\pi_P + 2\pi_R$), is as follows

$$TW^* = \frac{416 + 2432\gamma + 6076\gamma^2 + 8212\gamma^3 + 6267\gamma^4 + 2514\gamma^5 + 396\gamma^6}{8(2+\gamma)^2(1+2\gamma)(4+9\gamma+6\gamma^2)^2},$$

and

$$TW^c = \frac{104 + 232\gamma + 236\gamma^2 + 124\gamma^3 + 35\gamma^4 - 2\gamma^5}{8(2+\gamma)^4(1+2\gamma)}.$$

Taking the difference gives

$$TW^c - TW^* = -\frac{3\gamma(16 + 46\gamma - 46\gamma^2 - 395\gamma^3 - 749\gamma^4 - 696\gamma^5 - 324\gamma^6 - 54\gamma^7 + 6\gamma^8)}{2(2+\gamma)^4(1+2\gamma)(4+9\gamma+6\gamma^2)^2} > 0 \iff \gamma > \gamma^{TW},$$

with $\gamma^{TW} \simeq 0.33$ — i.e., $\gamma^{TW} \in (\gamma^{CS}, \hat{\gamma})$. □

[40] It can be easily checked that third-party sellers always benefit from colluding:

$$\pi_R^c - \pi_R^* = \frac{\gamma(1-\gamma)(8 + 61\gamma + 210\gamma^2 + 415\gamma^3 + 506\gamma^4 + 384\gamma^5 + 171\gamma^6 + 36\gamma^7)}{(2+\gamma)^4(1+2\gamma)(4+9\gamma+6\gamma^2)^2} > 0 \quad \forall \gamma \in (0,1).$$

C Ad-Valorem Fees

In this appendix, I analyse the game under the assumption that P charges each third-party seller R_i an ad-valorem (rather than a unit) fee $\phi_i \in [0,1]$ — i.e., R_i's profit is given by

$$\pi_{R_i} \triangleq ((1-\phi_i)p_i - c)D^i\big(p_i, p_j + p_P\big),$$

whereas P's profit is

$$\pi_P \triangleq \sum_{i=A,B} \phi_i p_i D^i\big(p_i, p_j + p_P\big) + (p_P - c)D^P\big(p_P, p_i + p_j\big),$$

where $c \in (0,1)$ is the unit production cost.

Non-cooperative Equilibrium. Maximizing firms' profits in stage $t = 2$ yields the pricing strategies

$$p_i(\phi_i; p_j^e, p_P^e) = \frac{1}{2}\left(\frac{c}{1-\phi_i} + \frac{1+\gamma(p_j^e + p_P^e - 1)}{1+\gamma}\right),$$

and

$$p_P(\phi_i, \phi_j; p_i^e, p_j^e) = \frac{1 + c(1+\gamma) + \gamma(p_i^e(1+\phi_i) + p_j^e(1+\phi_j) - 1)}{2(1+\gamma)}.$$

Proceeding as in the baseline model, it is easy to find that, absent collusion, in the negotiation between P and R_i, both players anticipate that, in the final price-setting stage, they will set prices

$$p_i^*(\phi_i, \phi^*) = \frac{c(1+\gamma)(2+(3-\phi_i)\gamma) + (1-\phi_i)(2+\gamma+2\gamma p^* - \gamma^2(3-(3+\phi^*)p^*))}{(1-\phi_i)(4+\gamma(8+(3-\phi_i)\gamma))},$$

and

$$p_P^*(\phi_i, \phi^*) = \frac{c(1+\gamma)(2+3\gamma-\phi_i(2+\gamma)) + (1-\phi_i)((1-\gamma)(2+(3+\phi_i)\gamma) + \gamma(2+(3+\phi_i)\gamma+2\phi^*(1+\gamma))p^*)}{(1-\phi_i)(4+\gamma(8+(3-\phi_i)\gamma))},$$

where $\phi_j^e = \phi^*$ is the (equilibrium) contract they expect it is offered to R_j, and p^* the (equilibrium) price R_j is supposed to set accordingly.

Solving P's problem in $t = 2$:

$$\max_{\phi_i} \phi_i D^i\big(p_i^*(\cdot), p^* + p_P^*(\cdot)\big) + \phi^* D^j\big(p^*, p_i^*(\cdot) + p_P^*(\cdot)\big) + p_P^*(\cdot) D^P\big(p_P^*(\cdot), p_i^*(\cdot) + p^*\big),$$

and imposing symmetry (i.e., $\phi_i = \phi^*$ and $p^* = p_i(\phi^*, \phi^*)$) yields the equilibrium fee, from which also $p_P^* = p_P(\phi^*, \phi^*)$ can be computed. However, it is not possible to find a closed-form solution, so that one must resort to numerical simulations to obtain the equilibrium values as functions of the parameters γ and c.

Collusive Equilibrium. Under collusion, maximizing the third-party sellers' joint profit yields

$$p(\phi; p_P^e) = \frac{1}{2}\left(1 + \frac{c}{1-\phi} - \gamma(1-p_P^e)\right),$$

whereas P's pricing strategy is as in the previous case, for $\phi_i = \phi_j = \phi$ and, accordingly, $p_i^e = p_j^e$.

Proceeding as in the baseline model, it is easy to find the SPNE prices for any given fee ϕ:

$$p^c(\phi) = \frac{c(1+\gamma)(2+(1-\phi)) + \gamma + (1-\phi)(1-\gamma)(2+3\gamma)}{2(1-\phi)(2+\gamma(2-\gamma-\phi\gamma))},$$

and

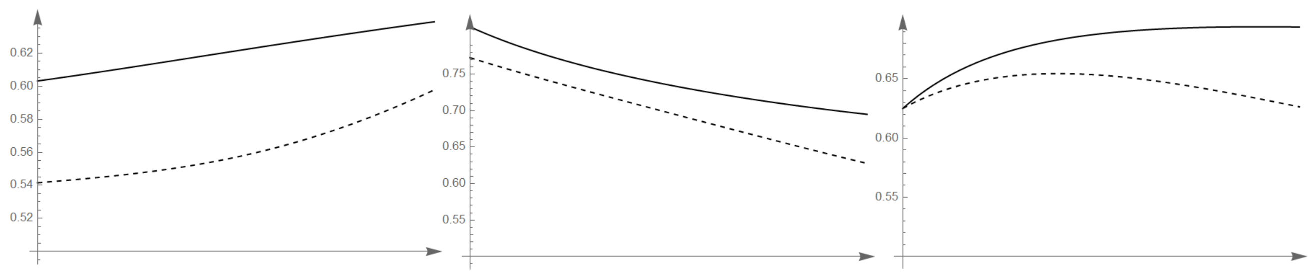
$$p_P^c(\phi) = \frac{c(1-\phi+2\gamma) + (1-\phi)(1-\gamma)(1+\gamma+\phi\gamma)}{(1-\phi)(2+\gamma(2-\gamma-\phi\gamma))}.$$

Solving P's problem in $t = 1$:

$$\max_{\phi} \sum_{i=A,B} \phi p^c(\phi) D^i\big(p^c(\phi), p^c(\phi) + p_P^c(\phi)\big) + (p_P^c(\phi) - c) D^P\big(p_P^c(\phi), 2p^c(\phi)\big),$$

yields the equilibrium fee ϕ^c, from which one can compute the equilibrium retail prices $p^c = p^c(\phi^c)$ and $p_P^c = p_P^c(\phi^c)$. Once again, the equilibrium can only be computed numerically.

Comparison. The comparison between equilibrium fees and prices in the two games is shown in Figure 1. The equilibrium values are plotted as functions of $\gamma \in (0,1)$ for a fixed value of c.[41]



(a) *Ad-valorem fees.* (b) *Price set by third-party sellers.* (c) *Price of P's private label.*

Figure 1: Equilibrium fees and prices in the non-cooperative equilibrium (continuous lines) and in the collusive equilibrium (dashed lines) as functions of $\gamma \in (0,1)$, for $c = 1/4$.

D Asymmetric Product Differentiation

In this appendix, I slightly change the demand system considered in the baseline model so to allow for asymmetric horizontal differentiation among products. Specifically, consider the following specification

[41]The numerical simulations from which these graphs are obtained were performed using Mathematica. The code is rather standard, hence it is not reported here, and is available upon request. Varying the value of c does not change the qualitative properties of the graphs. Further simulations are available upon request.

of the representative consumer's utility:

$$U(\cdot) \triangleq \sum_{h=A,B,P} q_h - \frac{1}{2}\sum_{h=A,B,P} q_h^2 - \gamma q_A q_B - \delta(q_A+q_B)q_P - \sum_{h=A,B,P} p_h q_h,$$

with $0<\delta\leq\gamma<1$. Assuming $\delta\leq\gamma$ entails that third-party sellers' products are closer substitutes of each other compared to P's private label, which is meant to make a case against collusion, as from the baseline analysis it is clear that the pro-competitive effects of collusion are magnified when P's private label is a close substitute of third-party sellers' products. Standard techniques then yield the (direct) demand functions

$$D^i\big(p_i, p_j+p_P\big) = \frac{(1-\delta)(1-\gamma)-(1-\delta^2)p_i+(\gamma-\delta^2)p_j+\delta(1-\gamma)p_P}{(1-2\delta^2+\gamma)(1-\gamma)}, \quad \forall i,j=A,B,\ j\neq i,$$

and

$$D^P\big(p_P, p_A+p_B\big) = \frac{1-2\delta+\gamma+\delta(p_A+p_B)-(1+\gamma)p_P}{1-2\delta^2+\gamma}.$$

By the same steps of the main analysis, it follows that the equilibrium without collusion is given by

$$\begin{aligned} f^* &= \frac{8(1+\gamma)-14\delta^2(1+\gamma)+6\delta^4(1+\gamma)-2\delta\gamma(1+\gamma)+\delta^3(1+3\gamma^2)}{2(6\delta^4(1+\gamma)+2(4-\gamma)(1+\gamma)-\delta^2(13+14\gamma-3\gamma^2))},\\ p^* &= \frac{6\delta^4(1+\gamma)+4(3-\gamma)(1+\gamma)-2\delta(2-\gamma)(1+\gamma)+\delta^3(3+(4-3\gamma)\gamma)-2\delta^2(8+3(3-\gamma)\gamma)}{2(6\delta^4(1+\gamma)+2(4-\gamma)(1+\gamma)-\delta^2(13+14\gamma-3\gamma^2))},\\ p_P^* &= \frac{1}{2}+\frac{2(1-\delta)^2\delta(1+\delta)}{6\delta^4(1+\gamma)+2(4-\gamma)(1+\gamma)-\delta^2(13+14\gamma-3\gamma^2)}, \end{aligned}$$

whereas, under third-party sellers' collusion,

$$f^c = \frac{4+\delta^3+4\gamma}{8+2\delta^2+8\gamma}, \qquad p^c = \frac{1}{4}\left(3-\delta+\frac{(1-\delta)\delta^2}{4+\delta^2+4\gamma}\right), \qquad p_P^c = \frac{1}{2}+\frac{\delta(1-\delta)}{4+\delta^2+4\gamma}.$$

Comparing the equilibrium prices, after some algebra, yields[42]

$$f^c < f^*, \quad p_P^c < p_P^*,$$

for all $0<\delta<\gamma<1$, and there are thresholds $\underline{\delta}\simeq 0.38$ and $\overline{\delta}\simeq 0.6$ such that $p^c<p^*$ if and only if:[43]

- $\delta\in(\underline{\delta},\overline{\delta})$ and $\gamma\in\left(\delta,\ \frac{-1+3\delta^2-3\delta^4+\sqrt{1+4\delta^2-2\delta^4-21\delta^6+18\delta^8}}{2-3\delta^2}\right)$; or
- $\delta\geq\overline{\delta}$ (for all $\gamma\geq\delta$).

[42] Detailed computations are omitted for brevity and available upon request.

[43] The threshold $\underline{\delta}$ is defined as the unique solution (in $(0,1)$) of $F(\delta)=\delta$, and $\overline{\delta}$ is defined as the unique solution (in $(0,1)$) of $F(\delta)=1$, with

$$F(\delta) = \frac{-1+3\delta^2-3\delta^4+\sqrt{1+4\delta^2-2\delta^4-21\delta^6+18\delta^8}}{2-3\delta^2}.$$

E Cost Asymmetry

In this appendix, I consider the game under the assumption that P suffers from a cost-disadvantage *vis-à-vis* third-party sellers — i.e., it incurs a unit production cost $c_P > 0$, so that its profit is

$$\pi_P \triangleq \sum_{i=A,B} f_i D^i\big(p_i, p_j + p_P\big) + (p_P - c_P) D^i\big(p_i, p_j + p_P\big),$$

with

$$c_P < \bar{c}_P = \frac{(1-\gamma)(2+3\gamma)(4+7\gamma+4\gamma^2)}{8+30\gamma+39\gamma^2+19\gamma^3} \in (0,1),$$

which guarantees that P sells a positive quantity in the equilibrium without third-party sellers' collusion and, *a fortiori*, in the equilibrium with collusion.

By the same steps of the baseline analysis, it can be shown that the equilibrium with competing third-party sellers is given by

$$f^* = \frac{8+\gamma(16-3\gamma^2(5+3\gamma)+c_P\gamma(2+3\gamma(1+\gamma)))}{2(1-\gamma)(2+\gamma)(4+9\gamma+6\gamma^2)},$$
$$p^* = \frac{12+\gamma(16-\gamma(6+\gamma)(1+3\gamma)+c_P(4+\gamma(6+\gamma-3\gamma^2)))}{2(1+\gamma)(2+\gamma)(4+9\gamma+6\gamma^2))},$$
$$p_P^* = \frac{(1-\gamma)(2+3\gamma)(4+\gamma(7+2\gamma))+c_P(8+\gamma(1-\gamma)(14+17\gamma+6\gamma^2))}{2(1-\gamma)(2+\gamma)(4+9\gamma+6\gamma^2)},$$

whereas, under third-party sellers' collusion,

$$f^c = \frac{4+4\gamma+(1-c_P)\gamma^3}{2(2+\gamma)^2}, \qquad p^c = \frac{6+\gamma(4-\gamma^2+c_P(2+2\gamma+\gamma^2))}{2(2+\gamma)^2}, \qquad p_P^c = \frac{1+c_P}{2} + \frac{\gamma(1-(1-c_P)\gamma)}{(2+\gamma)^2}.$$

Comparing equilibrium value yields

$$f^c - f^* = -\frac{(\gamma(1-(1-c_P)\gamma)(2+6\gamma+7\gamma^2-3\gamma^4)}{(1-\gamma)(2+\gamma)^2(4+9\gamma+6\gamma^2)} < 0,$$
$$p_P^c - p_P^* = -\frac{\gamma^2(1-(1-c_P)\gamma)(1+5\gamma+6\gamma^2)}{(1-\gamma)(2+\gamma)^2(4+9\gamma+6\gamma^2)} < 0,$$

for all $\gamma \in (0,1)$ and $c_P \in [0, \bar{c}_P)$, and

$$p^c - p^* = \frac{\gamma(1+\gamma)(1-(1-c_P)\gamma)(1-\gamma-3\gamma^2(1+\gamma))}{(1-\gamma)(2+\gamma)^2(4+9\gamma+6\gamma^2)} < 0 \iff \gamma > \hat{\gamma},$$

for all $c_P \in [0, \bar{c}_P)$.

These results immediately imply that $\gamma > \hat{\gamma}$ is a sufficient condition in order for collusion to be pro-competitive for all $c_P \in [0, \bar{c}_P)$. Moreover, comparing consumer surplus in the two equilibria gives

$$U^c - U^* > 0 \iff -16+\gamma(-22+164\gamma+613\gamma^2+784\gamma^3+187\gamma^4-570\gamma^5-690\gamma^6-360\gamma^7-90\gamma^8-c_P\Gamma) > 0,$$

with

$$\Gamma = (32 + 202\gamma + 558\gamma^2 + 766\gamma^3 + 362\gamma^4 - 372\gamma^5 - 648\gamma^6 - 378\gamma^7 - 90\gamma^8) < 0 \quad \forall \gamma \in (0,1),$$

which implies that, as c_P grows larger, collusion is more likely to be anti-competitive.

F $N \geq 2$ Third-Party Sellers

In this appendix, I develop the analysis assuming that there are $N \geq 2$ symmetric third-party sellers in the marketplace, competing with P's private label. The demand for each product $h = 1, \ldots, N, P$ is then defined as[44]

$$D^h\left(p_h, \sum_{k \neq h} p_k\right) \triangleq \frac{1-\gamma}{1+(N-1)\gamma - N\gamma^2}\left(1 - \frac{1+(N-1)\gamma}{1-\gamma}p_h + \frac{\gamma}{1-\gamma}\sum_{k=1,\ldots,N,P;\ k\neq h} p_k\right).$$

Non-cooperative Equilibrium. When all sellers compete with each other, as in the base model, in $t = 2$, each R_i, $i = 1, \ldots, N$, solves

$$\max_{p_i} (p_i - f_i) D^i\left(p_i, \sum_{k=1,\ldots N,P;\ k\neq i} p_k^e\right),$$

whereas P solves

$$\max_{p_P} \sum_{i=1,\ldots,N} f_i D^i\left(p_i^e, p_P + \sum_{j=1,\ldots N,\ j\neq i} p_j^e\right) + p_P D^P\left(p_P, \sum_{i=1,\ldots,N} p_i^e\right).$$

Thus, at the previous stage, by the Contract Equilibrium approach, when negotiations between P and R_i take place, both players believe that the other $N-1$ third-party sellers are facing the equilibrium fee f^*, and thus will set the equilibrium price p^*. Solving the above problems under these common beliefs and imposing that the corresponding FOCs are simultaneously satisfied yields

$$p_i^*(f_i, f^*) = \frac{2 + f_i(2 + 4\gamma(N-1) + \gamma^2(3 + 2N(N-2))) + \gamma(2N - 3 + 2(N-1)p^* + \gamma(1 + f^*(N-1) - 2N + p^* + N(2N-3)p^*))}{(2+\gamma(2N-3))(2+\gamma(2N-1))},$$

and

$$p_P^*(f_i, f^*) = \frac{2 + \gamma(-3 + \gamma + 3f_i(1+\gamma(N-1)) + 2f^*(1+\gamma(N-1))(N-1) + 2N - 2\gamma N + (N-1)(2 - \gamma + 2\gamma N)p^*)}{(2+\gamma(2N-3))(2+\gamma(2N-1))}.$$

Then, P's problem *vis-à-vis* R_i in $t = 1$ writes as

$$\max_{f_i} f_i D^i\big(p_i^*(\cdot), (N-1)p^* + p_P^*(\cdot)\big) + (N-1)f^* D^j\big(p^*, (N-2)p^* + p_i^*(\cdot) + p_P^*(\cdot)\big) + p_P^*(\cdot) D^P\big(p_P^*(\cdot), (N-1)p^* + p_i^*(\cdot)\big).$$

[44] These demand functions are derived from the following linear-quadratic utility function *à la* Singh and Vives (1984) of a representative consumer

$$U(\cdot) \triangleq \sum_{h=1,\ldots,N,P} q_h - \frac{1}{2}\sum_{h=1,\ldots,N,P} q_h^2 - \gamma \sum_{k,h=1,\ldots N,P;\ k\neq h} q_k q_h - \sum_{h=1,\ldots,N,P} p_h q_h.$$

Solving this problem and imposing symmetry (i.e., $f_i = f^*$ and $p^* = p_i^*(f^*, f^*)$) yields the equilibrium values

$$f^* = \frac{(2+\gamma(2N-1))(4+\gamma(8N-10+\gamma(7-10N+4N^2)))}{2(8+\gamma(22(N-1)+\gamma(21+6\gamma(N-1)^3+20N(N-2))))},$$
$$p^* = \frac{12+\gamma(4(8N-9)+\gamma(38+4N(7N-16)+\gamma(-13+4N(8-7N+2N^2))))}{2(8+\gamma(22(N-1)+\gamma(21+6\gamma(N-1)^3+20N(N-2))))},$$

thereby

$$p_P^* = p_P^*(f^*, f^*) = \frac{(2+\gamma(2N-1))(4+\gamma(8N-9+2\gamma(N-1)(2N-3)))}{2(8+\gamma(22(N-1)+\gamma(21+6\gamma(N-1)^3+20N(N-2))))}.$$

Collusive Equilibrium. Suppose now that all sellers form a cartel. Then, given the fee f negotiated with P in the first stage, the cartel solves

$$\max_{p_1,\ldots,p_N} \sum_{i=1,\ldots,N} (p_i - f)D^i\Bigg(p_i, \sum_{j=1,\ldots,N;\ j\neq i} p_j + p_P^e\Bigg).$$

Taking the FOC with respect to p_i and imposing symmetry yields each R_i's strategy $p_i = p(f; p_P^e)$ for all $i = 1, \ldots N$. Anticipating that all third-party sellers set the same price for any f, P's problem in $t = 2$ can be written as

$$\max_{p_P} NfD^i\big(p^e, (N-1)p^e + p_P\big) + p_P D^P\big(p_P, Np^e\big),$$

whose solution is denoted by $p_P = p_P(f; p^e)$. Using these two strategies it is easy to find the SPNE prices for any given fee f:

$$p^c(f) = \frac{(2+2f-\gamma)(1-\gamma)+\gamma(2-2\gamma+f(2+\gamma))N}{4-\gamma(4-(4-\gamma)N)}, \quad p_P^c(f) = \frac{2-\gamma(2-(1+3f-\gamma)N)}{4-\gamma(4-(4-\gamma)N)}.$$

Then, P's problem in the first stage writes as

$$\max_{f} NfD^i\big(p^c(f), (N-1)p^c(f) + p_P^c(f)\big) + p_P^c(f)D^P\big(p_P^c(f), Np^c(f)\big).$$

Solving this problem yields the equilibrium fee

$$f^c = \frac{8+8\gamma(N-1)+\gamma^3 N}{16-2\gamma(8-(8+\gamma)N)},$$

from which one can compute the equilibrium prices

$$p^c = p^c(f^c) = \frac{12-4\gamma(4-\gamma)+\gamma(12-2\gamma-\gamma^2)N}{16-2\gamma(8-(8+\gamma)N)}, \qquad p_P^c = p_P^c(f^c) = \frac{8-\gamma(8-(10-\gamma)N)}{16-2\gamma(8-(8+\gamma)N)}.$$

Comparison. Equilibrium fees and prices compare as follows:

$$f^c - f^* = -\frac{\gamma(N-1)(1-\gamma+\gamma N)(\gamma^2(8+4\gamma-3\gamma^2)N^2+\gamma(4-\gamma)(1-\gamma)(4+3\gamma)N+4(2-\gamma)(1-\gamma))}{(8-8\gamma+8\gamma N+\gamma^2 N)(8+\gamma(22(N-1)+\gamma(21+6\gamma(N-1)^3+20N(N-2))))} < 0,$$

and

$$p_P^c - p_P^* = -\frac{\gamma^2 N(N-1)(\gamma^2(2+7\gamma)N^2+2\gamma(1-\gamma)(2+7\gamma)N+(1-\gamma)(2+5\gamma-6\gamma^2))}{(8-8\gamma+\gamma(8+\gamma)N)(8+\gamma(22(N-1)+\gamma(21+6\gamma(N-1)^3+20N(N-2))))} < 0,$$

for all $\gamma \in (0,1)$ and $N \geq 2$, whereas

$$p^c - p^* = \frac{\gamma(N-1)(1-\gamma+\gamma N)(\gamma^2(4-10\gamma-3\gamma^2)N^2+\gamma(1-\gamma)(8-20\gamma-3\gamma^2)N+2(1-\gamma)(1-2\gamma)(2-3\gamma))}{(8-8\gamma+8\gamma N+\gamma^2 N)(8+\gamma(22(N-1)+\gamma(21+6\gamma(N-1)^3+20N(N-2))))}$$

is negative if and only if $\gamma > \hat{\gamma}$, for all $N \geq 2$.

These results immediately imply that there is a threshold $\gamma_N^{CS} \in (0, \hat{\gamma})$ such that collusion is consumer surplus increasing if and only if $\gamma > \gamma_N^{CS}$, with γ_N^{CS} being increasing in N. This is because, as N grows larger, the overall market share of third-party sellers increases, thereby, for all $\gamma < \hat{\gamma}$, relatively more consumers (i.e., those who purchase the more expensive products sold by third-party sellers) are harmed by collusion.

G Alternative Demand Specification

In this appendix, I show the robustness of the qualitative results under a different demand specification. Namely, I consider again $N \geq 2$ symmetric third-party sellers and a demand system *à la* Shubik and Levitan (1980):[45]

$$D^h\left(p_h, \sum_{k\neq h} p_k\right) \triangleq \frac{1}{N+1}\left[1 - \frac{1+(1+\mu)N}{N+1}p_h + \frac{\mu}{N+1}\sum_{k\neq h} p_k\right],$$

for each product $h = 1, \ldots, N, P$, with $k = 1, \ldots, N, P,\ k \neq h$, and $\mu \in (0, \infty)$ being an inverse measure of product differentiation.

Under this demand specification, equilibrium fees and prices when third-party sellers compete with

[45] These demand functions are derived from the following linear-quadratic utility function of a representative consumer

$$U(\cdot) \triangleq \sum_{h=1,\ldots,N,P} q_i - \frac{N+1}{2(1+\mu)}\left[\sum_{h=1,\ldots,N,P} q_i^2 + \frac{\mu}{N+1}\left(\sum_{h=1,\ldots,N,P} q_i\right)^2\right].$$

each other are given by

$$f^* = \frac{8+\mu^3+24(1+\mu)N+24(1+\mu)^2N^2+8(1+\mu)^3N^3}{2(8+2\mu+\mu^2+\mu^3+(4+\mu)(6+5\mu)N+2(1+\mu)(12+11\mu+\mu^2)N^2+2(1+\mu)^2(4+3\mu)N^3)},$$
$$p^* = \frac{32\mu N(1+N)^2+12(1+N)^3+2\mu^2(1+N)(1+14N^2)+\mu^3(1+8N^3)}{2(8+2\mu+\mu^2+\mu^3+(4+\mu)(6+5\mu)N+2(1+\mu)(12+11\mu+\mu^2)N^2+2(1+\mu)^2(4+3\mu)N^3)},$$
$$p_P^* = \frac{(2+\mu+2(1+\mu)N)(4+\mu(\mu-1)+8N+\mu(7-2\mu)N+4(1+\mu)^2N^2)}{2(8+2\mu+\mu^2+\mu^3+(4+\mu)(6+5\mu)N+2(1+\mu)(12+11\mu+\mu^2)N^2+2(1+\mu)^2(4+3\mu)N^3)}.$$

By contrast, the equilibrium values under third-party sellers' collusion are

$$f^c = \frac{1}{2}\left(1-\frac{\mu^2N(1+N)}{(1+\mu+N)(9\mu^2N+8(1+N)^2+8\mu(1+N)^2)}\right),$$
$$p^c = \frac{1}{4}\left(3-\frac{2\mu}{1+\mu+N}+\frac{\mu(8+(8+9\mu)N)}{9\mu^2N+8(1+N)^2+8\mu(1+N)^2}\right),$$
$$p_P^c = \frac{1}{2}+\frac{\mu N(1+N)}{9\mu^2N+8(1+N)^2+8\mu(1+N)^2}.$$

Comparing the two equilibria yields

$$f^c-f^* = -\frac{\mu(N-1)(1+N+\mu N)(9\mu^4N^2+8(1+N)^4+4\mu(1+N)^3(5+4N)+\mu^3(1+N)(1+4N)(4+5N)+8\mu^2(1+N)^2(2+5N+N^2))}{(1+\mu+N)(9\mu^2N+8(1+\mu)(1+N)^2)(8(1+N)^3+2\mu(1+N)^2(1+11N)+\mu^3(1+2N^2+6N^3)+\mu^2(1+N)(1+4N(1+5N)))},$$

and

$$p_P^c-p_P^* = -\frac{\mu^2N(N-1)(9\mu^3N^2+2(1+N)^3+\mu(1+N)^2(9+4N)+\mu^2(1+N)(1+2N(9+N)))}{(8+2\mu+\mu^2+\mu^3+(4+\mu)(6+5\mu)N+2(1+\mu)(12+11\mu+\mu^2)N^2+2(1+\mu)^2(4+3\mu)N^3)(9\mu^2N+8(1+N)^2+8\mu(1+N)^2)},$$

both expressions being negative for all $\mu > 0$ and $N \geq 2$, whereas

$$p^c-p^* = \frac{-\mu(N-1)(1+N+\mu N)(9\mu^4N^2-4(1+N)^4-2\mu(1+N)^3(4N-1)-4\mu^2(N+1)^2(N(N-1)-1)+\mu^3(N+1)(2N^2+15N-2))}{(1+\mu+N)(9\mu^2N+8(1+N)^2+8\mu(1+N)^2)(8(1+N)^3+2\mu(1+N)^2(1+11N)+\mu^3(1+2N^2+6N^3)+\mu^2(1+N)(1+4N(1+5N)))}$$

which is negative if and only if $\mu > \hat{\mu}_N$, with $\hat{\mu}_N > 0$ being increasing in N: see Figure 2. These results immediately imply that there is a threshold $\mu_N^{CS} \in (0, \hat{\mu}_N)$ such that collusion is consumer surplus increasing if and only if $\mu > \mu_N^{CS}$, with μ_N^{CS} being increasing in N.

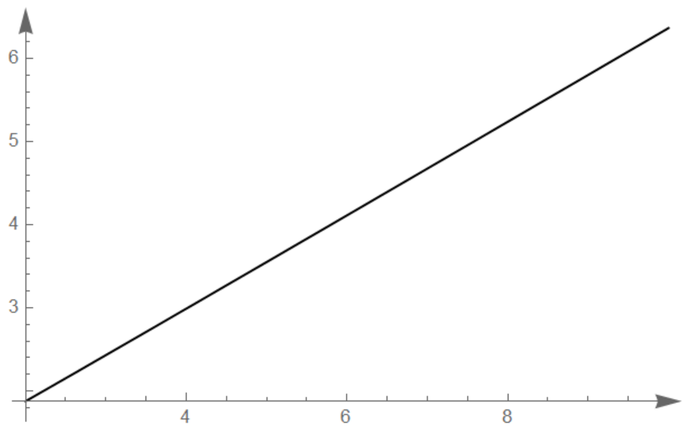


Figure 2: Threshold $\hat{\mu}_N$ as function of $N \in [2, 10]$.

H Alternative Timing and Wary Beliefs

In this appendix, I derive and compare the PBE with (affine) wary beliefs of the game under the alternative timing considered in Section 4.4.

Non-cooperative Equilibrium. Similar to the base model, each R_i's strategy, denoted by $\mathcal{P}_i(f_i)$, is defined as

$$\mathcal{P}_i(f_i) \triangleq \arg\max_{p_i}(p_i - f_i)D^i\big(p_i, \mathcal{P}_j(\mathcal{F}_j(f_i)) + \mathcal{P}_P(f_i)\big),$$

where $\mathcal{F}_j(\cdot)$ and $\mathcal{P}_P(\cdot)$ are P's optimal choices given f_i (defined below), and $\mathcal{P}_j(\cdot)$ is R_j's strategy.

Hence, in equilibrium, the following first-order condition must be satisfied:

$$D^i(\mathcal{P}_i(f_i), \mathcal{P}_j(\mathcal{F}_j(f_i)) + \mathcal{P}_P(f_i)) + (\mathcal{P}_i(f_i) - f_i)D^i_1(\cdot) = 0. \tag{7}$$

The functions $\mathcal{F}_j(f_i)$ and $\mathcal{P}_P(f_i)$ are then obtained as follows. By the logic of wary beliefs, upon observing f_i, R_i believes that P optimally sets f_j and p_P, given f_i — i.e.,

$$(\mathcal{F}_j(f_i), \mathcal{P}_P(f_i)) \triangleq \arg\max_{f_j, p_P} \sum_{h=i,j} f_h D^h(\mathcal{P}_h(f_h), \mathcal{P}_{-h}(f_{-h}) + p_P) + p_P D^P(p_P, \mathcal{P}_i(f_i) + \mathcal{P}_j(f_j)),$$

whose first-order conditions with respect to f_j and p_P imply that, in equilibrium:

$$D^j(\mathcal{P}_j(\mathcal{F}_j(f_i)), \mathcal{P}_i(f_i) + \mathcal{P}_P(f_i)) + \Big(f_i D^i_2(\cdot) + \mathcal{F}_j(f_i)D^j_1(\cdot) + \mathcal{P}_P(f_i)D^P_2(\cdot)\Big)\frac{\partial \mathcal{P}_j(\cdot)}{\partial f_j} = 0. \tag{8}$$

and

$$D^P(\mathcal{P}_P(f_i), \mathcal{P}_i(f_i) + \mathcal{P}_j(\mathcal{F}_j(f_i))) + \mathcal{P}_P(f_i)D^P_1(\cdot) + f_i D^i_2(\cdot) + \mathcal{F}_j(f_i)D^j_2(\cdot) = 0. \tag{9}$$

Following the literature (Rey and Vergé, 2004; Gaudin, 2019; Bisceglia, 2020), I consider symmetric equilibrium strategies of the form

$$\mathcal{P}(f) = \rho_0 + \rho_1 f, \qquad \mathcal{F}(f) = \theta_0 + \theta_1 f, \qquad \mathcal{P}_P(f) = \delta_0 + \delta_1 f.$$

Then, from the system of first-order conditions (7)-(8)-(9), proceeding by identification, it follows that ρ_1, θ_1 and δ_1 are the solution of

$$\begin{cases} 1 - 2\rho_1 + \gamma(1 + \delta_1 - \rho_1(2 - \theta_1)) = 0 \\ \delta_1\gamma(1 + \rho_1) - 2\rho_1(\theta_1 - \gamma(1 - \theta_1)) = 0 \\ \gamma(1 + \rho_1)(1 + \theta_1) - 2(1 + \gamma)\delta_1 = 0 \end{cases}$$

whereby they only depend on γ,[46] and the other parameters $\rho_0, \theta_0, \delta_0$ can then be obtained, again by

[46] This system can be solved using Mathematica, though the solution is very cumbersome (hence, omitted here).

identification, as function of ρ_1:

$$\rho_0 = \frac{4(1+2\gamma)(1-\gamma^2)\rho_1}{8\rho_1 - \gamma(\gamma - 22\rho_1 - \gamma\rho_1(11-2\rho_1) + \gamma^2(1+\rho_1)(1+3\rho_1))},$$

$$\theta_0 = \frac{(1-\gamma)(2+3\gamma)(1+\gamma+\gamma\rho_1)}{8\rho_1 - \gamma(\gamma - 22\rho_1 - \gamma\rho_1(11-2\rho_1) + \gamma^2(1+\rho_1)(1+3\rho_1))},$$

and

$$\delta_0 = \frac{\gamma(1-\gamma^2) + (1-\gamma)(2+3\gamma)^2\rho_1}{8\rho_1 - \gamma(\gamma - 22\rho_1 - \gamma(11-2\rho_1) + \gamma^2(1+\rho_1)(1+3\rho_1))}.$$

P's equilibrium choices are then obtained from

$$f^* = \mathcal{F}(f^*) \Longrightarrow f^* = \frac{\theta_0}{1-\theta_1},$$

and $p_P^* = \mathcal{P}_P(f^*)$, from which one can finally compute $p^* = \rho_0 + \rho_1 f^*$.

Collusive Equilibrium. The cartel, given fees f_A and f_B, set prices solving

$$\max_{p_A, p_B} \sum_{i=A,B} (p_i - f_i) D^i\big(p_i, p_j + \mathcal{P}_P(f_i, f_j)\big),$$

where $\mathcal{P}_P(\cdot)$ is P's optimal choice given the fees it offered to third-party sellers (see below). Solving the FOC of the cartel's problem yields prices

$$p_i^c(f_i, f_j) = \frac{1}{2}\left(1 + \gamma(\mathcal{P}_P(f_i, f_j) - 1) + f_i\right).$$

The function $\mathcal{P}_P(f_i, f_j)$ is defined as

$$\mathcal{P}_P(f_i, f_j) \triangleq \arg\max_{p_P} \sum_{i=A,B} f_i D^i\big(p_i^c(f_i, f_j), p_j^c(f_j, f_i) + p_P\big) + p_P D^P\big(p_P, p_i^c(f_i, f_j) + p_j^c(f_j, f_i)\big).$$

Taking the FOC then yields that, in equilibrium,

$$f_i D_2^i(\cdot) + f_j D_2^j(\cdot) + D^P(\mathcal{P}_P(f_i, f_j), p_i^c(f_i, f_j) + p_j^c(f_j, f_i)) + p_P D_1^P(\cdot) = 0.$$

As $D_2^i(\cdot) = D_2^j(\cdot)$, one can restrict attention to strategies of the form $\mathcal{P}_P(f_i, f_j) = \mathcal{P}_P(f_i + f_j)$. Moreover, following the literature, I consider again an affine strategy — i.e.,

$$\mathcal{P}_P(f_i + f_j) = \varphi_0 + \varphi_1(f_i + f_j).$$

Proceeding by identification, the above FOC yields

$$\varphi_0 = \frac{1-\gamma^2}{2+\gamma(2-\gamma)}, \qquad \varphi_1 = \frac{3\gamma}{4+2\gamma(2-\gamma)},$$

whereby the cartel's strategy rewrites as

$$p_i^c(f_i, f_j) = \frac{1}{2}\left(1 - \gamma + \frac{(2+\gamma)^2}{4 + 2\gamma(2-\gamma)\gamma} f_i + \frac{3\gamma^2}{4 + 2\gamma(2-\gamma)} f_j\right).$$

Using this strategy into P's problem at stage $t = 1$ and solving for f_A, f_B and p_P yields the symmetric equilibrium fee

$$f^c = \frac{4 + 4\gamma + \gamma^3}{2(2+\gamma)^2},$$

and the price of P's private label

$$p_P^c = \frac{4 + \gamma(6-\gamma)}{2(2+\gamma)^2}.$$

Finally, one can compute the price set by the cartel:

$$p^c = p^c(f^c, f^c) = \frac{6 + 4\gamma - \gamma^3}{2(2+\gamma)^2}.$$

Comparison. The comparison of equilibrium fees, prices and consumer surplus in the two games is shown in Figure 3. Hence, consumers benefit from third-party sellers' collusion if and only if $\gamma > \tilde{\gamma}^{CS} \simeq 0.61$, with $\tilde{\gamma}^{CS} > \gamma^{CS}$.

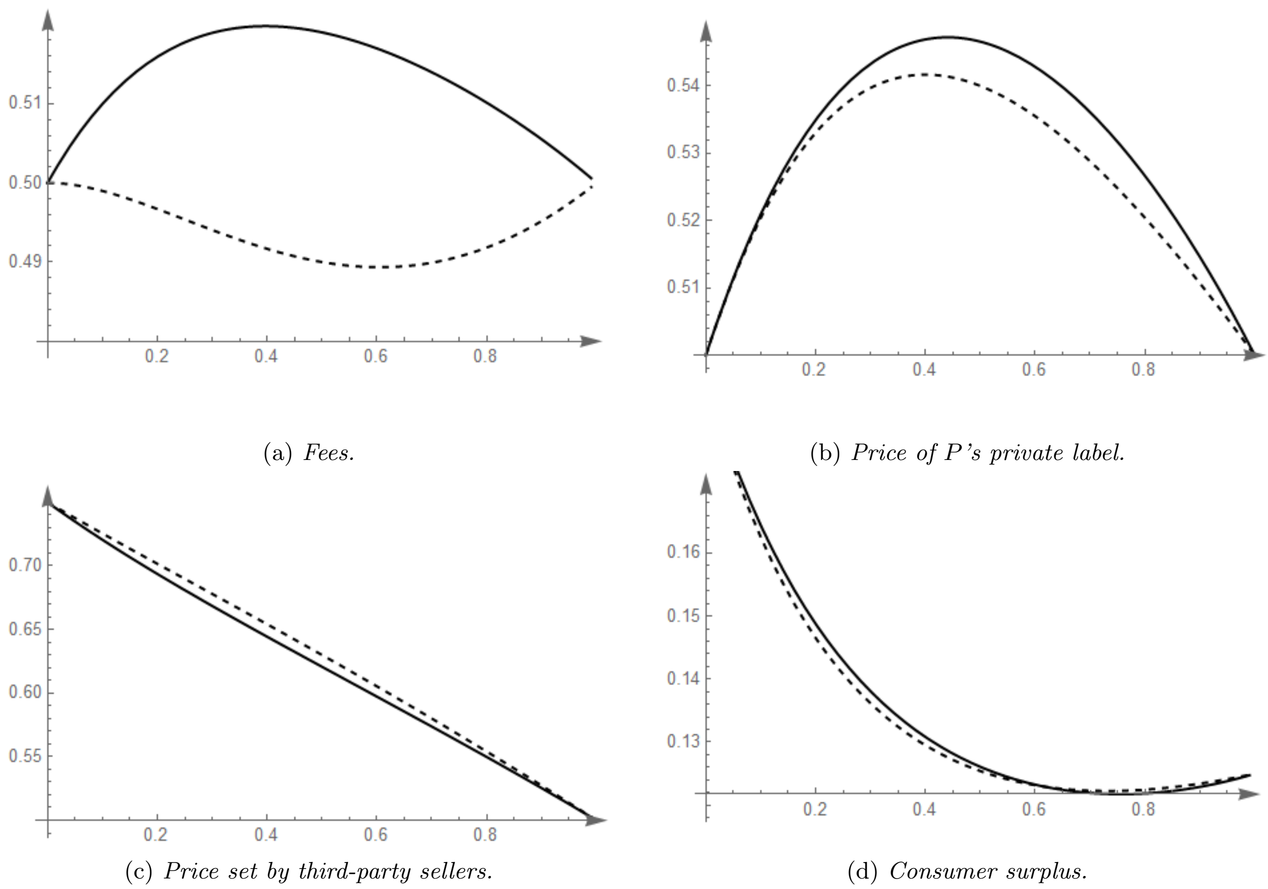


Figure 3: Equilibrium fees, prices and consumer surplus in the non-cooperative equilibrium (continuous lines) and in the collusive equilibrium (dashed lines) as functions of $\gamma \in (0, 1)$.

I Wholesale Business Model

In this appendix, I suppose that P adopts a wholesale business model. Specifically, consider the following timing of the game:

$t = 1$ Retailers R_A and R_B (non-cooperatively or cooperatively) set wholesale prices (w_A, w_B) for their products;

$t = 2$ P chooses retail prices p_A, p_B and p_P.

Hence, firms' profits are

$$\pi_{R_i} \triangleq w_i D^i\big(p_i, p_j + p_P\big),$$

and

$$\pi_P \triangleq \sum_{i=A,B} (p_i - w_i) D^i\big(p_i, p_j + p_P\big) + p_P D^P\big(p_P, p_A + p_B\big).$$

In this setting, regardless of R_A and R_B colluding or not, maximizing P's profit in $t = 2$ immediately yields

$$p_i = \frac{1 + w_i}{2}, \ \forall i = A, B, \qquad p_P = \frac{1}{2}.$$

Moving backward to the previous stage, when R_A and R_B compete with each other, each R_i solves

$$\max_{w_i} w_i D^i\left(\frac{1 + w_i}{2}, \frac{1 + w^*}{2} + \frac{1}{2}\right).$$

Solving this problem and imposing symmetry (i.e., $w_i = w^*$) gives

$$w^* = \frac{3}{2 + \gamma} - 1.$$

By contrast, the cartel solves

$$\max_{w_A, w_B} \sum_{i=A,B} w_i D^i\left(\frac{1 + w_i}{2}, \frac{1 + w_j}{2} + \frac{1}{2}\right),$$

whose solution is given by

$$w^c = \frac{1 - \gamma}{2}.$$

It can be immediately checked that $w^c > w^*$ for all $\gamma \in (0, 1)$ and, accordingly, $p^c > p^*$. Therefore, as $p_P = \frac{1}{2}$ in both games, collusion is always anti-competitive.

J *P* as a Pure Marketplace

In this appendix, I carry out the analysis under the assumption that all three products in the marketplace are sold by third-party sellers (denoted by R_A, R_B and R_C) — i.e., P is not vertically integrated, and just acts as the marketplace owner.

In this environment, it is straightforward to see that if all sellers collude, then collusion is always anti-competitive. In what follows, I show that collusion is always anti-competitive even if only two out of the three sellers, say R_A and R_B, form a cartel.

Non-Cooperative Equilibrium. Absent collusion, each R_i solves

$$\max_{p_i}(p_i - f_i)D^i\left(p_i, \sum_{j\neq i} p_j^e\right),$$

with $i, j = A, B, C, j \neq i$. In a symmetric equilibrium, $p_j^e = p^*$ for all j. The solution to this problem yields

$$p_i^*(f_i; p^*) = \frac{1}{2}f_i + \frac{1+\gamma(2p^* - 1)}{2(1+\gamma)}.$$

In $t = 1$, by the Contract Equilibrium approach, when offering a contract to R_i, P solves

$$\max_{f_i} f_i D^i\big(p_i^*(f_i; p^*), 2p^*\big) + \sum_{j\neq i} f^* D^j\big(p^*, p^* + p_i^*(f_i; p^*)\big).$$

Solving this problem and imposing symmetry (i.e., $f_i = f^*$ and $p_i^*(f^*; p^*) = p^*$) yields the equilibrium values

$$f^* = \frac{1+\gamma}{2+\gamma}, \qquad p^* = \frac{3+\gamma}{4+2\gamma}.$$

Collusive Equilibrium. Now I look for an equilibrium in which P offers a fee f^c to the cartel members (R_A and R_B) and f_C^c to R_C, and then both cartel members set the same price p^C, whereas R_C sets a price p_C^c.

The non-colluding retailer's problem is as in the previous case, thereby

$$p_C^c(f_C; p_A^e + p_B^e) = \frac{1}{2}f_C + \frac{1+\gamma(p_A^e + p_B^e - 1)}{2(1+\gamma)}.$$

By contrast, the cartel solves

$$\max_{p_A, p_B}(p_A - f)D^A\big(p_A, p_B + p_C^e\big) + (p_B - f)D^B\big(p_B, p_A + p_C^e\big),$$

where f is the fee P negotiates with the cartel. Solving the cartel's problem yields

$$p^c(f; p_C^e) = \frac{1}{2}(1 - \gamma(1 - p_C^e) + f).$$

Moving backward to the previous stage, by the Contract Equilibrium approach, when offering f to the cartel members, P solves

$$\max_{f} \sum_{i=A,B} f D^i\big(p^c(f; p_C^c), p^c(f; p_C^c) + p_C^c\big) + f_C^c D^C\big(p_C^c, 2p^c(f; p_C^c)\big),$$

whose solution is given by

$$f^c(f^c_C, p^c_C) = \frac{1}{2}(1 + \gamma(p^c_C + f^c_C - 1)).$$

Similarly, when negotiating with R_C, it solves

$$\max_{f_C} \sum_{i=A,B} f^c D^i\big(p^c, p^c + p^c_C(f_C; 2p^c)\big) + f_C D^C\big(p^c_C(f_C; 2p^c), 2p^c\big),$$

whose solution is given by

$$f^c_C(f^c, p^c) = \frac{1 - \gamma + 2\gamma(f^c + p^c)}{2(1+\gamma)}.$$

Using these strategies yields the equilibrium fees and prices of the cartel members,

$$f^c = \frac{(1+\gamma)(4+\gamma)}{8 + \gamma(8-\gamma)}, \qquad p^c = \frac{1}{2}\left(3 - \frac{12 + 13\gamma}{8 + \gamma(8-\gamma)}\right),$$

and of the other retailer R_C,

$$f^c_C = \frac{4 + 6\gamma}{8 + \gamma(8-\gamma)}, \qquad p^c_C = \frac{(6-\gamma)(1+\gamma)}{8 + \gamma(8-\gamma)}.$$

Comparison. The equilibrium fees and prices compare as follows:[47]

$$f^c < f^c_C < f^* \quad \forall \gamma \in (0, 1),$$

and

$$p^c > p^c_C > p^* \quad \forall \gamma \in (0, 1),$$

which immediately implies that collusion is always anti-competitive.

[47]Detailed computations are simple and omitted for brevity.